

A STRATEGIC APPROACH TO SOFTWARE PROTECTION*

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This paper demonstrates that there is a strategic reason why software firms have followed consumers' desire to drop software protection. We analyze software protection policies in a price-setting duopoly software industry selling differentiated software packages, where consumers' preference for particular software is affected by the number of other consumers who (legally or illegally) use the same software. Increasing network effects make software more attractive to consumers, thereby enabling firms to raise prices. However, it also generates a competitive effect resulting from fiercer competition for market shares. We show that when network effects are strong, unprotecting is an equilibrium for a noncooperative industry.

1. INTRODUCTION

Since the widespread introduction of personal computers in the early 1980s, software firms have gradually removed protection against copying. We see at least two reasons for this policy change on the part of firms. First, firms realized that consumers were annoyed by the protective devices, which compromised the effectiveness of their products.¹ Second, as we argue in this paper, when the market expands and competition intensifies, due to large network effects,

*We thank Hal Varian, two anonymous referees, and a coeditor for useful comments on earlier drafts.

1. For example, see announcements made by MicroPro International Corp. to drop the copy protection from WordStar 2000 in order to eliminate hardware incompatibility problems and simplify the installation procedure (*PC Week*, February 19, 1985), and by Ashton-Tate to immediately end copy protection on its most popular Dbase program (*Computerworld*, August 25, 1986).

firms have strategic incentives to remove protection in order to increase the number of consumers using their packages. Specifically, we explicitly address the issue of price competition in a differentiated software industry in which firms can choose whether to make their software easy to copy or prohibitively costly to copy. We then study the strategic incentives for firms to protect or not to protect their software against piracy.

Our model rests on the assumption that the value of using a software package increases with the number of people who legally and illegally use the same package. There are several empirical studies confirming the existence of software-specific network effects (see, e.g., Brynjolfsson and Kemerer, 1996, and Gandal, 1994). For example, Gandal finds that users of spreadsheet software highly value Lotus file compatibility. In the same vein, Brynjolfsson and Kemerer suggest that network-externality-type variables play an important role in the determination of software prices.

As observed by Conner and Rumelt (1991), piracy has two economic effects on software firms. First, piracy leads to a fall in direct sales. However, by increasing the size of the installed base, it may also boost the demand for the particular software. In this respect, Givon et al. (1995) report that pirates generated about 80% of the unit sales of spreadsheets and word processors in the UK. Installing protection in software has therefore two opposite effects, which have been analyzed by Conner and Rumelt in a monopoly setting. They found that, absent any network externality, a monopoly software developer increases price and profit when the exogenously chosen protection technology increases software protection. In contrast, when network externalities are present, profit can rise or fall as the level of piracy protection is increased.

The goal of our paper is to investigate related issues by introducing price competition among firms producing differentiated software packages. We demonstrate that protection can be used strategically, since protection removal enhances clientele just like strategic price cutting. In order to accomplish this analysis, we graft the network-externality model onto the Hotelling-type spatial competition model. In addition, we consider two groups of consumers: those who need the services provided by the software suppliers, and those who do not (support-independent consumers). For simplicity we assume that by protecting, firms can fully prevent all consumers from pirating their software.

Our main results are as follows. First, when firms protect their software, a low-price equilibrium emerges if network effects are

strong, whereas a high-price equilibrium arises under weak network effects. Therefore, all firms are better off with software protection when network effects are weak. In contrast, firms prefer not to protect their software when network effects are strong. The next set of results deals with a market situation where firms choose to protect or not, prior to price competition. For very weak network effects, both firms choose to protect their software because the impact of piracy on sales is insignificant. For intermediate value of the network effects, one firm chooses to protect whereas the other does not. This is because the network effects are now strong enough to induce one firm not to protect, thereby benefiting from the larger network size, whereas these effects are still too low for the other firm to be able to afford to do it. Furthermore, the nonprotecting firm earns a higher profit than the protecting firm. This suggests that the nonprotecting firm, because of its network size, builds a large network formed not only by pirates but also by legal users. Finally, our main result shows that, when network effects are sufficiently strong, both firms choose non-protection, since such a policy is now associated with large network sizes, consequent high consumers' valuations, and high profit levels. This result extends the monopoly result obtained by Conner and Rumelt (1991) to the case of a multistrategic oligopoly.

The literature on copying focuses on markets with no network effects, thereby making their analyses more applicable to journal, book, and music copying than to software (see Novos and Waldman, 1984; Johnson, 1985; Liebowitz, 1985; and Besen and Kirby, 1989). These papers show that even if consumer preferences for journals and books do not exhibit network externalities, publishers may still earn higher profits when photocopying of originals is allowed. In this case, restrictions on photocopying may reduce total welfare. These results were obtained under the assumption that publishes can price-discriminate between individual subscribers and libraries (or other types of dealers), by charging the libraries higher subscription rates that take into account the number of photocopies normally made from these journals. More precisely, the argument relies on the assumption that a library's willingness to pay for journals should increase when photocopying is done on the premises because the availability of photocopying causes library users to value the library's journal holdings more highly so that library funding will increase accordingly. Thus, these papers model the market for legal subscribers and photocopying as a market for durable goods, where photocopying is modeled as similar to a secondary market for used durable goods. In contrast, our paper provides an alternative approach to the literature

by ignoring the issue of appropriability of value from copies, and focusing instead on network effects.²

Besen and Kirby (1989) summarize these models and argue that the differences in conclusions regarding the effects of private copying on social welfare result from differences in (1) the extent to which the sellers of originals can appropriate the value placed on them by *all* users, (2) the relative market sizes for used and new copies, and (3) the degree of substitution between originals and copies. In the present paper we depart from the literature in two ways. First, we introduce price competition. Second, instead of focusing on appropriability, we introduce users' network externalities and heterogeneity across consumers with respect to the level of utility they derive from the support offered by software firms to their legal customers. Hence, one can say that one of the contributions of the present paper is that it provides a rational, other than the ability to appropriate, for firms to make copying/pirating easy.

A natural question to ask is why software piracy differs from journal and book photocopying, or even audio- and video-cassette duplication. Pirating software differs from journal and book photocopying in several aspects.

1. When software is not protected, any copy and copies of copies will be identical to the original. In contrast, paper and cassette copies are not equal to the originals, and copies of copies tend to be unreadable. Moreover, paper copying always loses information such as fine lines, fine print, and color images (even in color copying).
2. Therefore, in the case of photocopying, the number of copies made depends on the number of originals purchased in the market, whereas software piracy can potentially originate from a single diskette.
3. Journal and book publishers find it difficult and costly to physically protect their rights against illegal photocopying, whereas software developers can install protective devices that make piracy very difficult, and sometimes impossible.
4. Software users depend on services and documentation provided by developers, whereas copied journal articles and books can be

2. Consequently, our paper does not focus on the cost of duplication (assumed to be negligible for software) as a factor determining the ratio of copies to originals. Instead, we concentrate on the service provided by software firm to legal users.

read without reference to the original publishers. Similarly, listening and viewing audio and video cassettes does not require the use of any operating instructions from the manufacturer.

Because of these differences, the law treats photocopying and software piracy in different ways. For example, Section 170 of Copyright Act states: "... the fair use of copyrighted work... for purposes such as criticism, comment, newsreporting, teaching (including multiple copies for classroom use), scholarship, or research, is not an infringement of copyright." In contrast the Computer Software Copyright Act does not have the equivalent fair-use doctrine. Therefore, the law recognizes that the market consequences of photocopying for journal and book publishers are different from those of software piracy. For this reason, we limit the scope of this paper to analyzing the software industry.

The paper is organized as follows. Section 2 develops a duopoly model for the software industry where consumers' value of a software package increases with the number of other consumers using the same software. Section 3 solves for equilibrium software prices when firms do not protect their software. Section 4 solves for equilibrium when firms protect their software. Section 5 investigates the conditions under which software protection yields higher or lower industry profit than nonprotection. Section 6 analyzes market configurations where firms follow different protection policies. Section 7 investigates the conditions under which protection or nonprotection constitutes an equilibrium in a noncooperative software industry and whether the software industry benefits from the imposition of an industry-wide protection policy. Section 8 concludes.

2. A MODEL OF THE SOFTWARE INDUSTRY

Consider an industry with two firms producing two differentiated software packages denoted by A and B located at the endpoints of the interval $[0, 1]$. Let p_A denote the price of software package A and p_B the price of software package B . We assume that production is costless.

2.1. SOFTWARE USERS

Consumers are heterogeneous in two respects. First, some consumers gain extra utility from the services and support provided by the software firms to those customers who pay for the software, whereas

other consumers are support-independent and do not.³ Second, consumers rank the two software packages differently.

Formally, consumers are classified as:

- *Support-oriented consumers (type 1)*, who gain an extra utility $\sigma > 0$ from services and support provided by software firms to their legal customers. The ideal software packages of the support-oriented consumers are uniformly distributed over the interval $[0, 1]$. Thus, a consumer indexed by a high x is software-*B*-oriented, whereas a consumer indexed by a low x is software-*A*-oriented.
- *Support-independent consumers (type 2)*, who do not derive utility from the services and support provided by the software firms to their legal customers. The support-independent consumers are also uniformly distributed over the interval $[0, 1]$. Whenever it is convenient, we will index these consumers by y (rather than x) to distinguish between the two types.

The total population in the economy has a measure of 2. Hence we suppose that the populations of support-oriented and support-independent consumers have the same size; though restrictive, this assumption allows us to concentrate on the pure effect of competition on the strategic choices made by firms regarding software protection. This assumption is relaxed in the concluding section.

Each consumer in the economy has five options: the consumer can buy software *A*, buy software *B*, pirate software *A*, pirate software *B*, or not use any software. In case of pirating, the consumer does not pay for the software and does not receive any support from software firms.

ASSUMPTION 1: Software firms bundle the support with purchase. Illegal software users cannot obtain support from an independent supplier.

Let n_A (similarly, n_B) denote the number of consumers who *legally and illegally* use software *A* (software *B*). We assume that consumers' utility is enhanced with an increase in the number of other consumers using (legally or illegally) the same software package. The assumption of a network externality here means that consumers benefit from exchanging files generated by the same software

3. This distinction is similar to the distinction in the copying literature between the relative value of copies and originals to different consumers. For example, support-oriented consumers could also be those who are strongly risk-averse with respect to being prosecuted for using software illegally.

packages and that files generated by different software are incompatible.⁴

Thus, the utility of a consumer of type $i = 1, 2$ and indexed by $x \in [0, 1]$ is given by

$$U(x, i) \equiv \begin{cases} -x + \mu n_A - p_A + s_i & \text{if buys software } A, \\ -x + \mu n_A & \text{if pirates software } A, \\ -(1-x) + \mu n_B - p_B + s_i & \text{if buys software } B, \\ -(1-x) + \mu n_B & \text{if pirates software } B, \\ 0 & \text{if does not use software,} \end{cases}$$

where $s_i \equiv \begin{cases} \sigma, & i = 1, \\ 0 & i = 2, \end{cases}$ (1)

where $\mu \geq 0$ is the coefficient measuring the importance of the network size to a software user.

The utility function (1) implies that a support-oriented consumer will prefer buying software A instead of pirating software A if and only if $\sigma \geq p_A$, that is, if the utility from the customer support provided by firm A is larger than the package's price. Similarly, a support-oriented consumer would prefer buying software B over pirating software B if and only if $\sigma \geq p_B$.

We will use the following notation. For a given price pair (p_A, p_B) , let \hat{x}_A be the support-oriented consumer who is indifferent between buying software A and not buying any software. Formally, \hat{x}_A is the solution to $U(\hat{x}_A, 1) = -\hat{x}_A + \mu n_A - p_A + \sigma = 0$. \hat{x}_B is similarly defined. Let \hat{y}_A be the support-independent consumer who is indifferent between pirating software A and not using any software. Formally, \hat{y}_A is the solution to $U(\hat{y}_A, 2) = -\hat{y}_A + \mu n_A = 0$. \hat{y}_B is similarly defined. Finally, let \hat{x} be the support-oriented consumer indifferent between software A and B . Formally \hat{x} solves $-x + \mu n_A - p_A + \sigma = -(1-x) + \mu n_B - p_B + \sigma$, or

$$\hat{x} = \frac{1 + \mu(n_A - n_B) + p_B - p_A}{2}. \tag{2}$$

2.2. SOFTWARE INDUSTRY EQUILIBRIUM

Since consumers' value of a particular software package increases with the number of people using it, we model the market as a two-stage game in which both firms and consumers are players. The

4. Whereas the introduction of variable compatibility would make the model more realistic, Chou and Shy (1993) show that partial compatibility generates severe discontinuity modeling problems.

solution concept used is the subgame-perfect Nash equilibrium. In the first stage, firms select their software prices $p_i \in [0, \infty)$. In the second stage, given any pair of prices, p_A and p_B , potential software users make adoption decisions. A software adoption equilibrium of a second-stage subgame is a partition of consumers between those who buy software A (B), those who pirate software A (B), and nonusers, such that no individual whose utility is specified in (1) would be strictly better by changing his adoption or nonadoption behavior.

The proof of the following lemma is given in Appendix A.

LEMMA 1: *Let p_A and p_B be any pair of prices satisfying $p_A, p_B \leq \sigma$. If $\mu < \frac{1}{2}$, then there is an adoption equilibrium such that all support-oriented consumers buy software.*

However, when both p_A and p_B are large enough, there exists a second adoption equilibrium, which turns out to be unstable. This equilibrium involves some support-oriented consumers who do not buy and do not pirate any software. We analyze this equilibrium for software A only. It is described by the following conditions:

$$-\hat{x}_A - p_A + \mu n_A + \sigma = 0, \quad -\hat{y}_A + \mu n_A = 0, \quad \text{and}$$

$$n_A = \hat{x}_A + \hat{y}_A,$$

which are solved for

$$\hat{x}_A = \frac{1 - \mu}{1 - 2\mu}(\sigma - p_A),$$

which is smaller than $\frac{1}{2}$ as long as p_A is close enough to σ . This equilibrium is unstable because slightly increasing (decreasing) the number of A users leads to an increase (decrease) in A 's network size, thereby increasing (decreasing) both the number of support-oriented consumers buying software A and the number of support-independent users pirating this software. Note that this instability is generated by marginal deviation of support-oriented and/or support-independent consumers. Hence, there exists a unique stable equilibrium such that the entire support-oriented population is served, whereas the second equilibrium is unlikely to be realized.

In what follows we focus only on the stable adoption equilibrium. Then, firms' profits are defined as the number of consumers *buying* their software times their price (recall that the number of buyers can be smaller than the number of users, since some users may pirate the software). In the first stage, we solve for a Nash

equilibrium where both firms simultaneously choose their prices so as to maximize their profit.

We make the following assumptions.

ASSUMPTION 2: The network-effect parameter is bounded: $\mu < \frac{1}{2}$.

If Assumption 2 is reversed, then there does not exist a pure-strategy Nash equilibrium in software prices in which both firms sell strictly positive amounts and earn strictly positive profits. In fact, when network effects are very strong, each firm wants to undercut its rival's price by subsidizing the "transportation" cost of the consumer most oriented toward its rival, thereby gaining a larger network of consumers.

ASSUMPTION 3: The support-oriented consumers place a high value on the support they can receive from software firms. Formally, $\sigma > \frac{3}{2}$.

This assumption allows us to restrict the number of market configurations to be investigated in that only the support-independent consumers may find it optimal to opt out.

In the next two sections we first describe consumers' behavior and then solve for equilibrium prices when neither firm protects its software and when both firms protect their software.

3. EQUILIBRIUM PRICES WHEN FIRMS DO NOT PROTECT THEIR SOFTWARE

Suppose that neither firm protects its software; hence each consumer can either buy the software (and obtain support, if needed), or can costlessly pirate and use the software (without obtaining support).

It follows from the utility functions given in (1) that no consumer will purchase software i if $p_i > \sigma$, since the software's price exceeds the support-oriented consumers' utility from the service provided by the software firms to legal users. In this case all users will prefer pirating software over buying it. Hence, in equilibrium it must be that software firms set $p_i \leq \sigma$, $i = A, B$. Therefore, (1) implies that support-oriented consumers never pirate software. Among the support-oriented consumers, we know that the consumer who is indifferent between buying software A and buying software B is given by

$$\hat{x} = \frac{1 + \mu(n_A - n_B) + p_B - p_A}{2}$$

whose location is depicted in the upper part of Figure 1. Notice that the location of the marginal consumer is affected not only by the relative software prices, $p_B - p_A$, but also by the difference in network sizes, $n_A - n_B$.

As shown in the following lemma, the utility function (1) and Assumption 2 imply, with a zero reservation utility, that some support-independent consumers will not use any software even if they can obtain it illegally for free (the proof is given in Appendix B).

LEMMA 2: *When neither firm protects its software, (a) some support-independent users pirate software A and some pirate software B, and (b) some support-independent consumers do not use any software.*

The consequences of Lemma 2 are illustrated in the bottom part of Figure 1, where some (but not all) of the support-independent consumers pirate software. Recall that \hat{y}_A (\hat{y}_B) denotes the support-independent consumer who is indifferent between pirating software A (software B) and not using any software. Therefore,

$$\hat{y}_A = \mu n_A \quad \text{and} \quad \hat{y}_B = 1 - \mu n_B. \tag{3}$$

For the consumer partition depicted in Figure 1 to constitute an adoption equilibrium, the numbers of A and B (legal and illegal) users are implicitly given by

$$n_A = \hat{x} + \hat{y}_A + \frac{1 - \mu n_B - p_A + p_B}{2 - 3\mu},$$

$$n_B = (1 - \hat{x}) + (1 - \hat{y}_B) = \frac{1 - \mu n_A - p_B + p_A}{2 - 3\mu}.$$

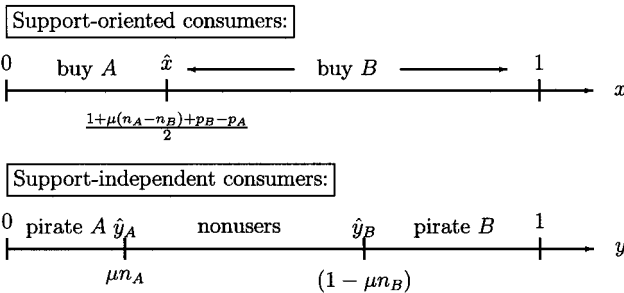


FIGURE 1. TOP: THE SUPPORT-ORIENTED CONSUMER, \hat{x} , WHO IS INDIFFERENT BETWEEN BUYING A-SOFTWARE AND B-SOFTWARE; BOTTOM: THE SUPPORT-INDEPENDENT CONSUMER, \hat{y}_A (\hat{y}_B), WHO IS INDIFFERENT BETWEEN PIRATING SOFTWARE A (SOFTWARE B) AND NOT USING ANY SOFTWARE.

Solving for n_A and n_B yields

$$n_A = \frac{\mu(p_A - p_B - 2) - p_A + p_B + 1}{2(2\mu^2 - 3\mu + 1)} \quad \text{and} \quad (4)$$

$$n_B = \frac{\mu(p_B - p_A - 2) + p_A - p_B + 1}{2(2\mu^2 - 3\mu + 1)}.$$

Substituting (4) into (2), we have

$$\hat{x}(p_A, p_B) = \frac{\mu(p_A - p_B - 2) - p_A + p_B + 1}{2(1 - 2\mu)}. \quad (5)$$

We now look for a Nash equilibrium in software prices in which firm A chooses p_A to maximize $\pi_A = p_A \hat{x}(p_A, p_B)$ and firm B chooses p_B to maximize $\pi_B = p_B [1 - \hat{x}(p_A, p_B)]$, where $\hat{x}(p_A, p_B)$ is given in (5). The best-response functions are given by

$$p_A = R_A(p_B) = \frac{1 - 2\mu}{2(1 - \mu)} + \frac{p_B}{2} \quad \text{if } p_A < \sigma,$$

$$p_B = R_B(p_A) = \frac{1 - 2\mu}{2(1 - \mu)} + \frac{p_A}{2} \quad \text{if } p_B < \sigma. \quad (6)$$

The equilibrium prices and profit levels when both firms do not protect are given by

$$p_A^u = p_B^u = \frac{1 - 2\mu}{1 - \mu} > 0 \quad \text{and} \quad \pi_A^u = \pi_B^u = \frac{1 - 2\mu}{2(1 - \mu)} > 0. \quad (7)$$

Using Assumption 3, it can be checked that the equilibrium prices are smaller than σ , thereby satisfying the two best-response functions (6). Substituting (7) into (4) yields

$$n_A^u = n_B^u = \frac{1}{2(1 - \mu)} > \frac{1}{2}, \quad (8)$$

implying that some support-independent consumers pirate software. To find the number of consumers pirating software A and B , we

subtract the number of legal users from (8). Therefore,

$$y_A^u = 1 - y_B^u = \frac{1}{2(1 - \mu)} - \frac{1}{2} = \frac{\mu}{2(1 - \mu)} < \frac{1}{2}.$$

Consequently, we have shown:

PROPOSITION 1: *When software is unprotected, a unique equilibrium exists for any admissible value of μ .*

4. EQUILIBRIUM PRICES WHEN FIRMS PROTECT THEIR SOFTWARE

We now suppose that each software firm possesses the means of protecting their software packages, thereby making software piracy not beneficial to any consumer. For example, each software firm may set the software so that a special plug or a chip is necessary to launch the application. Then consumers must choose between buying the software and not using any software. In order to highlight the strategic importance of protection, we assume that software protection is costless for the software firms (see also Conner and Rumelt, 1991).

Lemma 2 shows that not all support-independent consumers pirate software when software is unprotected. Therefore, when software is protected, it must be that some support-independent consumers do not purchase any software. Consequently, we need to derive equilibrium prices for the two cases where (i) some (but not all) support-independent consumers buy software, and (ii) none of the support-independent consumers buy software.⁵

4.1. SOME SUPPORT-INDEPENDENT CONSUMERS PURCHASE SOFTWARE

The marginal support-oriented consumer is still given by (2). The support-independent consumer \hat{y}_A who is indifferent between buying Software *A* and not using any software is found by solving $U(y_A, 2) = -y_A + \mu n_A - p_A = 0$. Similarly, the support-independent consumer \hat{y}_B who is indifferent between purchasing software *B* and not using any software is found by solving $U(y_B, 2) = -(1 - y_B) + \mu n_B - p_B = 0$. Hence,

$$\hat{y}_A = \mu n_A - p_A \quad \text{and} \quad \hat{y}_B = 1 - \mu n_B + p_B. \quad (9)$$

5. Recall that we have seen in Section 2.2 that, for any price pair, there exists a unique stable adoption equilibrium, so that the first-stage profit functions are uniquely defined.

The number of *A*-software users (which equals the number of *A*-buyers, since software is protected) is $n_A = \hat{x} + \hat{y}_A$. The number of *B*-software users (buyers) is equal $n_B = (1 - \hat{x}) + (1 - \hat{y}_B)$. Substituting (2) and (9) into these equations and then solving simultaneously for n_A and n_B yields

$$n_A = \frac{2\mu(2p_A - 1) - 3p_A + p_B + 1}{2(2\mu^2 - 3\mu + 1)} \quad \text{and}$$

$$n_B = \frac{2\mu(2p_B - 1) - 3p_B + p_A + 1}{2(2\mu^2 - 3\mu + 1)}. \quad (10)$$

Since both software firms protect their software, the number of buyers equals the number of users of each software package. Therefore, firm *A* chooses p_A to maximize $\pi_A = p_A n_A$, and firm *B* chooses p_B to maximize $p_B n_B$, where n_A and n_B are given in (10). The best-response functions are given by

$$p_A = R_A(p_B) = \frac{1 - 2\mu + p_B}{2(3 - 4\mu)} \quad \text{if } p_A < \sigma,$$

$$p_B = R_B(p_A) = \frac{1 - 2\mu + p_A}{2(3 - 4\mu)} \quad \text{if } p_B < \sigma. \quad (11)$$

Therefore, if a Nash equilibrium exists, it must be that prices, numbers of buyers, and profit levels are given by

$$p_A^p = p_B^p = \frac{1 - 2\mu}{5 - 8\mu}, \quad n_A^p = n_B^p = \frac{3 - 4\mu}{2(1 - \mu)(5 - 8\mu)},$$

$$\pi_A^p = \pi_B^p = \frac{(1 - 2\mu)(3 - 4\mu)}{2(1 - \mu)(5 - 8\mu)^2}. \quad (12)$$

The numbers of support-independent consumers buying software *A* and software *B* are given by

$$\hat{y}_A^p = \mu n_A - p_A^p = \frac{8\mu^2 - 9\mu + 2}{2(1 - \mu)(8\mu - 5)} = 1 - \hat{y}_B^p$$

$$\geq 0 \quad \text{if and only if} \quad \mu > \frac{9 - \sqrt{17}}{16}.$$

Let $\mu_m \stackrel{\text{def}}{=} \frac{2}{5}$. The following proposition is proved in Appendix C.

PROPOSITION 2: *When software is protected, an equilibrium where some support-independent consumers buy software exists if and only if $\mu \geq \mu_m$.*

If $\mu < \mu_m$, the network effect is sufficiently weak to induce each firm to raise its price, thereby specializing upon support-oriented consumers only. In contrast, when $\mu \geq \mu_m$ protection leads to an increase in the number of buyers from both firms. This follows from the fact that no support-independent consumers buy software in the absence of protection. However, in spite of the increase in sales, comparing (7) and (12) reveals that firms make lower profits under protection. This is due to the fact that protection results here in a sharp drop in equilibrium prices, as shown by comparing (7) and (12).

4.2. SUPPORT-INDEPENDENT CONSUMERS DO NOT BUY SOFTWARE

We now solve for an equilibrium where software firms set high prices, so all support-independent consumers refrain from buying (and hence from using) any software. In this case, $n_A = \hat{x}$ and $n_B = 1 - \hat{x}$, where \hat{x} is given in (2). Solving these two equations for n_A and n_B yields

$$n_A = \frac{1 - \mu - p_A + p_B}{2(1 - \mu)} \quad \text{and} \quad n_B = \frac{1 - \mu - p_B + p_A}{2(1 - \mu)}.$$

Firm *A* chooses p_A to maximize $\pi_A = p_A n_A$, and firm *B* chooses p_B to maximize $\pi_B = p_B n_B$, yielding best-response functions $p_A = R_A(p_B) = (1 - \mu + p_B)/2$ and $p_B = R_B(p_A) = (1 - \mu + p_A)/2$. Hence, the candidate equilibrium prices, number of buyers, and profit levels are

$$p_A^p = p_B^p = 1 - \mu, \quad n_A^p = n_B^p = \frac{1}{2}, \quad \pi_A^p = \pi_B^p = \frac{1 - \mu}{2}. \quad (13)$$

We need to confirm that at these prices, none of the support-independent consumers buys any software. To see this, observe that the utility of the consumer indexed by $y = 0$ when buying software *A* is $U(0, 2) = -0 + \mu \times \frac{1}{2} - (1 - \mu) < 0$, since $\mu < \frac{1}{2}$.

Finally, in order for the prices (13) to constitute an equilibrium, no firm should be able to increase its profit by sharply reducing its price, thus attracting some of the support-independent consumers to

buy its software. Appendix D provides the proof for the following proposition. Let

$$\mu_M \stackrel{\text{def}}{=} \frac{5 - \sqrt{17}}{2}. \tag{14}$$

PROPOSITION 3: *When software is protected, an equilibrium where no support-independent consumers buy software exists if and only if $\mu \leq \mu_M$.*⁶

If the condition of the proposition is reversed, the network effect becomes so strong that each firm can increase its profit by unilaterally lowering its price, thereby making some support-independent consumers buying its software.

Comparing (7) and (13) reveals that firms now make higher profits under protection, because price competition is softened due to the weaker effect of smaller network sizes.

4.3. SUMMARY OF EQUILIBRIA WHEN BOTH FIRMS PROTECT

We have shown that, depending on the value of μ , when both firms protect their software so that piracy is not an option for consumers, two equilibria may exist: a low-price equilibrium where some service-independent consumers buy software, and a high-price equilibrium where service-independent consumers do not buy (and therefore do not use) any software. Figure 2 illustrates how the two equilibria are related to the network parameter μ .

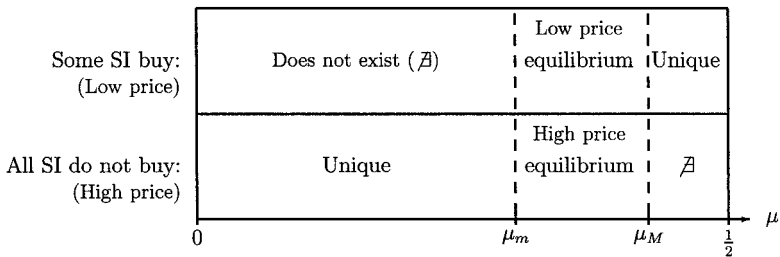


FIGURE 2. SUMMARY OF EQUILIBRIA WHEN BOTH FIRMS PROTECT THEIR SOFTWARE.

SI = support-independent consumers.

6. For $\mu = \mu_M$ there exist two equilibria.

5. SOFTWARE INDUSTRY'S PROTECTION POLICY

In this section, we analyze how software protection affects industry profit and software prices by comparing the two policies analyzed in Sections 3 and 4.

First, for $\mu \leq \mu_M$, comparing (7) and (8) with (13) yields

$$\begin{aligned} p^u - p^p &= \frac{\mu^2}{\mu - 1} < 0, & n^u - n^p &= \frac{\mu}{2(1 - \mu)} > 0, \\ \pi^u - \pi^p &= \frac{\mu^2}{2(\mu - 1)} < 0. \end{aligned} \tag{15}$$

Second, for $\mu \geq \mu_m$, comparing (7) and (8) with (12) yields

$$\begin{aligned} p^u - p^p &= \frac{(1 - 2\mu)(4 - 7\mu)}{(1 - \mu)(5 - 8\mu)} > 0, \\ n^u - n^p &= \frac{1 - 2\mu}{(1 - \mu)(5 - 8\mu)} > 0, \\ \pi^u - \pi^p &= \frac{(1 - 2\mu)^2(11 - 16\mu)}{(1 - \mu)(5 - 8\mu)^2} > 0. \end{aligned} \tag{16}$$

Last, prices and profits are higher in (13) than in (12). These results lead to the following proposition.

PROPOSITION 4:

1. *There are more (buying plus pirating) software users when firms do not protect than when firms protect their software.*
2. *Let $0 < \mu \leq \mu_m$. Then firms' prices and profit levels are higher when both firms protect their software.*
3. *Let $\mu_m < \mu \leq \mu_M$. Then profits are higher under protection at the high-price equilibrium, and lower at the low-price equilibrium, than profits under nonprotection.*
4. *Let $\mu_M < \mu < \frac{1}{2}$. Then firms' prices and profit levels are higher when firms do not protect their software.*

The intuition behind Proposition 4 is as follows. For small values of μ ($\mu \leq \mu_m$), the network effect is weak and the sole buyers are the support-oriented consumers. Hence, the price-competition effect dominates the network effect and both firms are better off by protecting, since this allows them to relax price competition in a

market of a given size. In contrast, for *large* values of μ ($\mu > \mu_M$), the network effect is stronger than the competition effect, so that both firms gain by expanding the network of users. Although firms could expand the number of legal users by protecting the software, they earn higher profits by not protecting, because they are able to charge a much higher price to the support-oriented consumers.

Finally, for the intermediate values of μ (belonging to a domain of size smaller than 0.04), it is hard to predict what is the optimal industry policy, since it depends on the particular equilibrium that will arise under protection. However, since for $\mu_m < \mu \leq \mu_M$ the high-price equilibrium under protection dominates both the equilibrium without protection and the low-price equilibrium under protection from the firms' viewpoint, it is reasonable to suppose that minimal coordination will take place within the industry, leading firms to select the high-price equilibrium together with the protection policy.

Altogether, we may conclude that *it is in the interest of the software industry to implement nonprotection when network effects are strong*, while protection is preferable otherwise. Though empirical evidence is missing, the first scenario might well be the more likely one for the software industry.

6. EQUILIBRIUM PRICES WHEN FIRM **A** PROTECTS AND FIRM **B** DOES NOT PROTECT

In order to study a noncooperative software industry where firms are free to choose their own protection policy, we need to derive equilibrium prices when firms use different protection policies. With no loss of generality, suppose that firm *A* protects its software whereas firm *B* does not. In this case, similarly to the analysis of Section 4, there can be two equilibria: one in which some service-independent consumers purchase software *A* (the protected software) and a second one where the price of *A*-software is high, so that service-independent consumers do not purchase software *A*.

6.1. SOME SUPPORT-INDEPENDENT CONSUMERS PURCHASE SOFTWARE

Let $\hat{y}_A > 0$. Then the number of support-independent consumers buying software *A* is given by (9), so that $n_A = \hat{x} + \hat{y}_A$. Similarly, the number of support-independent consumers pirating software *B* can be obtained from (3), so that $n_B = 1 - \hat{x} + 1 - \hat{y}_B$. Substituting for \hat{x}

into these equations and solving simultaneously for n_A and n_B yields

$$n_A = \hat{x} + \hat{y}_A = \frac{1 - 2\mu + (4\mu - 3)p_A + (1 - \mu)p_B}{2(2\mu^2 - 3\mu + 1)},$$

$$n_B = 1 - \hat{x} + 1 - \hat{y}_B = \frac{1 - 2\mu + p_A + (\mu - 1)p_B}{2(2\mu^2 - 3\mu + 1)}.$$

Firm A chooses p_A to maximize $\pi_A = p_A n_A$, and firm B chooses p_B to maximize $\pi_B = p_B(1 - \hat{x})$. Solving the first-order conditions yields the prices

$$p_A^p = \frac{3(2\mu - 1)}{16\mu - 11} \quad \text{and} \quad p_B^u = \frac{16\mu^2 - 22\mu + 7}{(\mu - 1)(16\mu - 11)}. \quad (17)$$

Hence, the numbers of users of each software package are

$$n_A^p = \frac{3(4\mu - 3)}{2(1 - \mu)(16\mu - 11)} \quad \text{and} \quad n_B^u = \frac{8\mu - 7}{2(1 - \mu)(16\mu - 11)}.$$

It is readily verified that the corresponding value of \hat{y}_A is positive if and only if $\mu > (9 - \sqrt{17})/16$. Finally, the profit levels are given by

$$\pi_A^p = \frac{9(2\mu - 1)(4\mu - 3)}{2(1 - \mu)(16\mu - 11)^2} \quad \text{and} \quad (18)$$

$$\pi_B^u = \frac{(8\mu - 7)(16\mu^2 - 22\mu + 7)}{2(1 - \mu)(16\mu - 11)^2}.$$

It remains to check under which conditions firm A does not find it profitable to raise its price and to serve only the support-oriented consumers. The following proposition is proven in Appendix E.

PROPOSITION 5: *If $\mu \geq \mu_m$, then (17) constitutes a unique asymmetric price equilibrium.*

6.2. SUPPORT-INDEPENDENT CONSUMERS DO NOT PURCHASE SOFTWARE

When $\hat{y}_A = 0$, the number of software- A buyers (which equals the number of users) is $n_A = \hat{x}$, where \hat{x} is given in (2). The number of support-independent consumers who pirate software B is found from

$-(1 - \hat{x}) + \mu n_B = 0$, where \hat{x} is given in (2). Substituting (2) into these equations, and solving simultaneously for n_A and n_B yields

$$n_A^p = \hat{x} = \frac{1 - 2\mu - (1 - \mu)(p_A - p_B)}{\mu^2 - 4\mu + 2}$$

and

$$n_B^u = 1 - \hat{x} + 1 - \hat{y}_B = \frac{1 - \mu + p_A - p_B}{\mu^2 - 4\mu + 2}.$$

Firm A chooses p_A to maximize $\pi_A = p_A \hat{x}$, and firm B chooses p_B to maximize $\pi_B = p_B(1 - \hat{x})$, yielding the prices

$$p_A^p = \frac{\mu^2 - 6\mu + 3}{3(1 - \mu)} \quad \text{and} \quad p_B^u = \frac{2\mu^2 - 6\mu + 3}{3(1 - \mu)}. \tag{19}$$

Hence, the numbers of users are

$$n_A^p = \frac{\mu^2 - 6\mu + 3}{3(\mu^2 - 4\mu + 2)} \quad \text{and} \quad n_B^u = \frac{2\mu^2 - 6\mu + 3}{3(1 - \mu)(\mu^2 - 4\mu + 2)}.$$

It can now be easily verified that $\mu n_A^p - p_A^p < 0$; hence service-independent consumers do not purchase software A . Also, it can be shown that $n_B^u > \frac{1}{2}$ and that $\hat{x} > \frac{1}{2}$, which implies that some support-independent consumers pirate software B .

Finally, the profit levels are

$$\begin{aligned} \pi_A^p &= p_A^p \hat{x} = \frac{(\mu^2 - 6\mu + 3)^2}{9(1 - \mu)(\mu^2 - 4\mu + 2)}, \\ \pi_B^u &= p_B^u(1 - \hat{x}) = \frac{(2\mu^2 - 6\mu + 3)^2}{9(1 - \mu)(\mu^2 - 4\mu + 2)^2}. \end{aligned} \tag{20}$$

We now check under which conditions firm A will find it unprofitable to lower its price and to serve some support-independent consumers. Appendix F provides the proof for the following proposition.

PROPOSITION 6: *If $\mu \leq \mu_m$, then (19) constitutes a unique asymmetric price equilibrium.*

Equations (19) and (20) as well as (17) and (18) reveal that $p_B^u > p_A^p$ and $\pi_B^u > \pi_A^p$ regardless of the value of μ . In words, for any degree of network effect, the unprotecting firm charges the higher price and earns a larger profit. The intuition is that, due to the network effects, the firm that does not follow a protection policy can charge a higher price because its software is used by more consumers, and hence is more valuable to some support-oriented consumers. Despite the fact that this firm has a smaller number of buyers than its rival ($\hat{x} > \frac{1}{2}$), it earns a higher profit.

7. SOFTWARE PROTECTION STRATEGIES

So far, we have investigated the effects of software protection assuming that firms follow the same policy regarding protection. In this section, we investigate a noncooperative software industry where each firm is free to choose its own protection policy. To this end, we add a preliminary stage in which both firms simultaneously choose from the two-action set {U, P}, where U stands for not protecting and P for protecting.

In the remainder of the paper, we ignore the small parameter range $\mu_m < \mu < \mu_M$ in order to limit the number of cases to investigate and to focus upon low or high network effects only. It is our belief that not much relevant information is lost by making this assumption. We will use the following terminology.

DEFINITION 1: We say that network effects are *weak* if $\mu < \mu_m$ and *strong* if $\mu > \mu_M$.

7.1. EQUILIBRIUM PROTECTION POLICIES UNDER WEAK NETWORK EFFECTS

Suppose that $\mu < \mu_m$. Table I provides the profit levels of software firms A and B for the four possible outcomes, given in (7), (13), and (20).

Direct calculations from Table I yield the following result.

PROPOSITION 7: *When network effects are weak,*

1. *if $\mu < 0.2765$, both firms protecting their software, (P, P), constitutes a unique Nash equilibrium;*
2. *If $\mu \geq 0.2765$, there are exactly two Nash equilibria, (P, U) and (U, P), where one firm protects its software and the other does not.*

Thus, when the network effects are very weak, an industry-wide protection policy is supported as a Nash equilibrium. For stronger

TABLE I.
EQUILIBRIUM PROFITS UNDER WEAK NETWORK EFFECTS

		Firm B	
		P	U
Firm A	P	$\frac{1 - \mu}{2}$	$\frac{1 - \mu}{2}$
	U	$\frac{(2\mu^2 - 6\mu + 3)^2}{9(1 - \mu)(\mu^2 - 4\mu + 2)^2}$	$\frac{(\mu^2 - 6\mu + 3)^2}{9(1 - \mu)(\mu^2 - 4\mu + 2)}$
		$\frac{(\mu^2 - 6\mu + 3)^2}{9(1 - \mu)(\mu^2 - 4\mu + 2)}$	$\frac{1 - 2\mu}{2(1 - \mu)}$
		$\frac{1 - 2\mu}{2(1 - \mu)}$	$\frac{1 - 2\mu}{2(1 - \mu)}$

but still moderate network effects, asymmetric protection policies are the only equilibria and they do not support collusion.

7.2. EQUILIBRIUM PROTECTION POLICIES UNDER STRONG NETWORK EFFECTS

Suppose that $\mu > \mu_M$. Table II provides the profit levels of software firms A and B for the four possible outcomes, given in (7), (12), and (18).

Direct calculations from Table II yield the following result.

PROPOSITION 8: *When network effects are strong, there are exactly two equilibria, (P, P) and (U, U), where both firms protect or both refrain from protecting their software.*

An important conclusion that we draw from this proposition is that a mutual decision to protect or not to protect software can be enforced as a noncooperative outcome. As shown by Proposition 4, (U, U) yields strictly higher profits to both firms than (P, P), so that it is reasonable to assume that (U, U) will prevail. Consequently, the foregoing result provides a rationale why software firms have complied with consumers' desires to remove protection from software packages since the mid-1980s. Our result also shows that not protecting can be sustained as a Nash equilibrium of the protection game when network effects become sufficiently strong, something that seems to have happened as computers gradually entered our daily routine.

7.3. SEQUENTIAL CHOICE OF PROTECTION POLICIES

As suggested by a referee, it is worthwhile to investigate a decision-making process in which one firm chooses its protection policy before its rival, while prices are simultaneously chosen only after both firms have selected their protection policies.

Under sequential moves, Proposition 7 remains unchanged except for part 2, where (U, P) is a unique equilibrium, since the firm that is first to choose its protection policy will choose not to protect, as that yields larger profits (see discussion following Proposition 6).

On the other hand, Proposition 8 is modified in that (U, U) is the only equilibrium outcome, since it yields a higher industry profit and therefore the first mover will pick U. This additional result highlights the fact that, for strong network effects, nonprotection is the unique equilibrium outcome.

TABLE II.
EQUILIBRIUM UNDER STRONG NETWORK EFFECTS

		Firm B	
		P	U
Firm A	P	$\frac{(1 - 2\mu)(3 - 4\mu)}{2(1 - \mu)(5 - 8\mu)^2}$	$\frac{(1 - 2\mu)(3 - 4\mu)}{2(1 - \mu)(5 - 8\mu)^2}$
	U	$\frac{(8\mu - 7)(16\mu^2 - 22\mu + 7)}{2(1 - \mu)(16\mu - 11)^2}$	$\frac{9(2\mu - 1)(4\mu - 3)}{2(1 - \mu)(16\mu - 11)^2}$
		$\frac{1 - 2\mu}{2(1 - \mu)}$	$\frac{1 - 2\mu}{2(1 - \mu)}$
		$\frac{(8\mu - 7)(16\mu^2 - 22\mu + 7)}{2(1 - \mu)(16\mu - 11)^2}$	$\frac{(8\mu - 7)(16\mu^2 - 22\mu + 7)}{2(1 - \mu)(16\mu - 11)^2}$

8. CONCLUDING REMARKS

The paper analyzes a trade-off faced by competing software firms. Each firm can increase the competitive value of its software by not protecting it. Alternatively, each firm can protect its software by reducing the number of users to the number of buyers, thus making its software less attractive. Proposition 4 demonstrates that a coordinated software industry should choose not to protect the software when the network effects are strong. The reason is that a larger number of users increases the utility of software. Thus, the paper provides a strategic reason why the use of software protection has declined since the mid-1980s.

Our results were derived under the assumption that the numbers of support-oriented and support-independent consumers are the same and equal to one. One may wonder how our results would be affected when there are fewer support-independent than support-oriented consumers. In order to gain some insight, we consider the extreme case in which there are no support-independent consumers. In this case, it is readily verified that the equilibrium profits are $\pi_A = \pi_B = (1 - \mu)/2$, which are exactly the equilibrium profits given in (13) when network effects are not strong and both firms protect. This is because under the high-price equilibrium support-independent consumer do not buy the software, thereby making their market immaterial. On the other hand, when network effects are strong, $\pi_A = \pi_B = (1 - \mu)/2$ can no longer be obtained in equilibrium, since price competition is very intense due to the stronger network effects in the presence of support-independent consumers.

This discussion leads to the following important conclusion: when network effects are not strong, protecting is equivalent to the nonexistence of support-independent consumers. When network effects are strong, that is no longer so. Indeed, in this case we have shown that firms prefer not to protect their software. Altogether, under strong network effects firms are harmed by the existence of support-independent consumers, and we conjecture that they become worse off as the relative number of support-independent consumers rises.

APPENDIX A. PROOF OF LEMMA 1

The support-oriented consumer who is indifferent between software A and B is

$$\hat{x} = \frac{p_B - p_A + \mu(n_A - n_B) + 1}{2}. \quad (21)$$

Since $n_A = \hat{x} + \hat{y}_A$ and $\hat{y}_A = \mu n_A$, we obtain

$$n_A = \frac{\hat{x}}{1 - \mu}. \quad (22)$$

Similarly, since $n_B = (1 + \hat{x}) + (1 - \hat{y}_B)$ and $1 - \hat{y}_B = \mu n_B$, we get

$$n_B = \frac{1 - \hat{x}}{1 - \mu}. \quad (23)$$

Substituting (22) and (23) into (21) yields

$$\hat{x} = \frac{1 - \mu}{2(1 - 2\mu)}(p_B - p_A) - \frac{1}{2}. \quad (24)$$

To prove the lemma, it remains to show that the utility of consumer \hat{x} is strictly positive. Substituting (24) into (22) and then into (1), some manipulations lead to

$$U(\hat{x}, 1) = \frac{1}{2} - \frac{\mu}{2(1 - \mu)} + \sigma - \frac{p_A + p_B}{2} > 0,$$

because $p_A, p_B \leq \sigma$ and $\mu < \frac{1}{2}$.

APPENDIX B. PROOF OF LEMMA 2

(a): Lemma 1 implies that, in equilibrium, all support-oriented consumers are served, so that $n_A + n_B \geq 1$. With no loss of generality, we can assume that $n_A \geq \frac{1}{2}$. By way of contradiction, suppose that none of the support-independent consumers pirate any software. Hence, the utility of the support-independent consumer indexed by $y = 0$, when pirating software A , is $U(0, 2) = -0 + \mu n_A > 0$, a contradiction.

(b): If all support-independent consumers pirate software, then it must be that $n_A + n_B = 2$. Consider the nondegenerate interval $(\mu n_A, \mu n_A + 1 - 2\mu)$ of the support-independent consumers. For any y in this interval, we have $y > \mu n_A$, so that $-y + \mu n_A < 0$, which implies that consumer y does not pirate software A . Similarly, we have $y < \mu n_A + 1 - 2\mu$, or equivalently, $-1 + y + 2\mu - \mu n_A < 0$, which in turn amounts to $-(1 - y) + \mu n_B < 0$, since $n_B = 2 - n_A$, so that consumer y does not want to pirate software B .

APPENDIX C. PROOF OF PROPOSITION 2

Suppose that firm B maintains its equilibrium price, $p_B = (1 - 2\mu)/(5 - 8\mu)$. We now check under what condition firm A cannot increase its profit by raising its price p_A , thereby losing its support-independent consumers. Substituting $p_B = (1 - 2\mu)/(5 - 8\mu)$ into (2) yields $\hat{x} = [\mu(n_A - n_B) - p_A + (1 - 2\mu)/(5 - 8\mu)]/2$. The number of A -users (A -buyers) is now $n_A = \hat{x}$. Substituting \hat{x} into this equation and solving for n_A yields

$$n_A = \frac{-2(8\mu^2 - 10\mu + 3) + (8\mu^2 - 13\mu + 5)p_A}{(\mu^2 - 4\mu + 2)(8\mu - 5)}.$$

Firm A chooses p_A to maximize $\pi_A = p_A \hat{x}$, yielding

$$p_A = \frac{8\mu^2 - 10\mu + 3}{(\mu - 1)(8\mu - 5)}, \quad \pi_A = \frac{(8\mu^2 - 10\mu + 3)^2}{(1 - \mu)(\mu^2 - 4\mu + 2)(8\mu - 5)^2}. \quad (25)$$

To find under which condition this deviation by firm A is not profitable, we check that the profit given (25) is smaller than or equal to the profit given in (12) if and only if $\mu \geq \frac{2}{5}$.

APPENDIX D. PROOF OF PROPOSITION 3

Suppose that firm B maintains its equilibrium price, $p_B = 1 - \mu$, given in (13). We now check under what condition firm A cannot increase its profit by lowering its price, p_A , thereby attracting some support-independent consumers to buy software A . Substituting $p_B = 1 - \mu$ into (2) yields $\hat{x} = [\mu(n_A - n_B) - p_A + 2 - \mu]/2$. The support-independent consumer who is indifferent between buying software A and not using any software is given by $\hat{y}_A = \mu n_A - p_A$. The number of A -users (A -buyers) is $n_A = \hat{x} + \hat{y}_A$. The number of B -users (B -buyers) is $n_B = 1 - \hat{x}$ (support-independent consumers do not purchase B -software at $p_B = 1 - \mu$). Substituting \hat{x} and \hat{y}_A into these equations and solving for n_A yields

$$n_A = \frac{2(1 - \mu) - (3 - \mu)p_A}{\mu^2 - 4\mu + 2}.$$

Firm A chooses P_A to maximize $\pi_A = p_A n_A$, yielding

$$p_A = \frac{1 - \mu}{3 - \mu}, \quad n_A = \frac{1 - \mu}{\mu^2 - 4\mu + 2}, \quad (26)$$

$$\pi_A = \frac{(1 - \mu)^2}{(3 - \mu)(\mu^2 - 4\mu + 2)}.$$

To find under which condition this deviation by firm A is not profitable, one can show that the profit given (26) is smaller than or equal to the profit given in (13) if and only if $\mu \leq (5 - \sqrt{17})/2$.

APPENDIX E. PROOF OF PROPOSITION 5

Consider a price deviation by firm A such that this firm serves only support-oriented consumers, that is, $\hat{y}_A = 0$. Substituting for p_B given in (17) into (2), we obtain

$$n_A = \hat{x} = \frac{-6(8\mu^2 - 10\mu + 3) + (16\mu^2 - 17\mu + 11)p_A}{(\mu^2 - 4\mu + 2)(16\mu - 11)}.$$

The maximum profit under deviation is then given by

$$\pi_A = \frac{9(8\mu^2 - 10\mu + 3)^2}{(1 - \mu)(\mu^2 - 4\mu + 2)(16\mu - 11)^2}. \quad (27)$$

Comparing (18) and (27) shows that deviation is not profitable if and only if $\mu \geq \mu_m$.

APPENDIX F. PROOF OF PROPOSITION 6

Consider a price deviation by firm A such that this firm serves some support-independent consumers, that is, $\hat{y}_A > 0$. In this case, we have

$$n_A = \hat{x} + \hat{y}_A = \frac{2(\mu^2 - 6\mu + 3) + 3(4\mu - 3)p_A}{6(2\mu^2 - 3\mu + 1)}.$$

The maximum profit under deviation is then given by

$$\pi_A = \frac{(\mu^2 - 6\mu + 3)^2}{18(3 - 4\mu)(2\mu^2 - 3\mu + 1)}. \quad (28)$$

Comparing (20) and (28) shows that deviation is not profitable if and only if $\mu \leq \mu_m$.

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