

PRICING A NETWORK GOOD TO DETER ENTRY*

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This paper develops a model of pricing to deter entry by a sole supplier of a network good. We show that the installed user base of a network good can serve a preemptive function similar to that of an investment in capacity if the entrant's good is incompatible with the incumbent's good and there are network externalities in demand. Consequently, the threat of entry can lead the incumbent to set low prices. We identify some factors that should be considered in thinking about the welfare effects of entry deterrence in this and similar models.

I. INTRODUCTION

BOTH SIDES IN *US v. MICROSOFT* seem to agree that Microsoft's pricing of Windows does not correspond to short-run profit maximization by a monopolist. Schmalensee's direct testimony argues that Microsoft's low prices are due at least in part to its concern that higher prices would encourage other firms to develop competing operating systems. While this idea may seem intuitive, it has been seen as controversial by some commentators (e.g. Hall and Hall [1999]) because neither side has proposed a formal model where such 'limit pricing' would make sense. In response, this paper develops a highly simplified model of complete-information limit pricing, based on the idea that the installed base of a network good can fill a preemptive role similar to that of investment in physical capacity.

To model the role of the incumbent's installed base, we assume that there are overlapping generations of consumers, each of whom lives for two periods and purchases only when young. We also assume there are only two types of consumers, and we further assume that the distribution of values is such that the incumbent's steady-state profit is highest when it

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sells to only the high-value types as opposed to all of them, so that market power without the threat of entry will lead to lower output than is socially optimal. These simplifying assumptions are not crucial for our conclusions, and we discuss more general setups at various places in the text.

To model entry, we suppose there is a sequence of potential entrants, each of whom hopes to enter the market and become the new incumbent. Although technological progress leads to steadily decreasing quality-adjusted prices, our assumptions will imply that the competitive environment of an entrant who successfully takes over the market is identical to the one that the incumbent would have faced, so that once an entrant captures the market it sets exactly the same price as its predecessor.¹ The entry cost of each entrant is random, and the date- t entrant learns its cost at the beginning of date t . It then decides whether or not to enter; if it chooses not to enter its payoff is zero and it leaves the game. We suppose that the entrant's good is better than, but incompatible with, the incumbent's product, so that if entry occurs purchasers face a tradeoff between inherent quality and network benefits.

Before proceeding to analyze the effects of entry and entry deterrence in this model, we first analyze the model when the incumbent is not faced with the threat of entry. Because the incumbent lacks commitment power, and the price that current consumers are willing to pay depends on their forecast of future network benefits, this problem has a non-trivial dynamic structure. Under the standard assumption that purchasers coordinate on the equilibrium that they all prefer, we show that the unique Markov-perfect equilibrium in the absence of an entry threat is to sell to only the high-value consumers.²

We then develop conditions under which the threat of entry leads the incumbent firm to set lower prices than it would otherwise have done in order to eliminate or reduce the probability of entry. In this equilibrium, each entrant enters *iff* its entry cost is less than a critical value k^* . Since this critical value will depend on the installed base for reasons we explain below, the incumbents will choose to sell to all consumers rather than just the high-value ones in order to reduce the probability of entry in the future. If the entrant does come in at date t , the resulting Bertrand competition leads it to capture the entire market in that period; the incumbent leaves at $t + 1$, and the entrant becomes the new incumbent.

¹ This sort of stationarity has been used in a number of past papers on entry deterrence, for example Eaton and Lipsey [1980]. In their model the extra investment used to deter entry always decreases welfare since it has no effect on price or production cost. In this respect our findings are closer to work on contestability, see e.g. the axiomatic treatment in Baumol *et al.* [1982] and the dynamic game studied by Maskin and Tirole [1988].

² Past studies of the pricing of network goods have either used a two-period model where expectations and dynamics play a smaller role, or supposed that all consumers have identical preferences, so that the monopoly pricing problem is trivial.

In the simplest version of the model, where prices (which are the same as markups because we assume that marginal cost is zero) can be negative, when entry occurs the incumbent sets a price sufficiently negative that it is just indifferent between exiting and remaining the incumbent. The entrant then sets its price at the level where consumers are just willing to purchase from it; this 'introductory offer' results in all consumers buying from the entrant. It also results in the entrant 'rebating' to consumers the discounted value of its future profits, so that the payoff to entry gross of entry costs is the extent to which current consumers prefer the entrant's product. This preference, which we call the entrant's 'efficiency advantage,' is the difference between the entrant's technological edge and the incumbent's greater network benefits. If prices are constrained to be non-negative, the entrant's payoff to entry is the sum of the efficiency advantage and a term that reflects 'rent-stealing' from the established incumbent. In either case, the payoff to entry is a decreasing function of the incumbent's installed base; this is why increasing the installed base reduces the probability of entry.

We analyze the welfare implications of these equilibria, and find that *in equilibrium* the welfare effects of inducing additional entry are ambiguous. Substantial insight into the comparison between the equilibrium and optimal amount of entry can be obtained by comparing the entrant's private return to entry with the social return. Since the entrant's payoff gross of entry costs corresponds to the increased utility it gives to date-*t* consumers, the difference between the private and social return comes from effects on consumers of other generations and (in the case of non-negative prices) from the reduction in the incumbent's rents. In particular, the entrant's private returns do not appropriate the welfare improvements coming from technological spill-overs in future generations, while the private return to entry ignores the welfare losses incurred by 'stranded users' who stick with the old standard. If there is a binding non-negativity constraint on prices, the private return to entry also includes 'rent-stealing,' which is another force that tends to promote excessive entry. To determine whether welfare would be improved by marginal increases or decreases in the equilibrium entry threshold, one must approximate the values of these three discrepancies between the private and social return.

Several previous papers have pointed out the social costs of stranded users. Farrell and Saloner [1986] analyzed the social cost of stranded users in a model where the incompatible technologies are competitively supplied. Katz and Shapiro [1992] is closer to this paper, because it assumes each technology is supplied by a single firm, and that Bertrand competition post-entry leads the entrant to capture the entire market at a price that is reduced by the size of the incumbent's network advantage. In their model, consumers live forever, and differ only in the date that they arrive on the market. Thus the incumbent sells to the entire inflow of new consumers

until entry occurs, just as it would do without the threat of entry, so there is no scope in the model for limit pricing. Since delaying entry reduces production cost, but increases the network handicap, the entrant faces a timing problem; Katz and Shapiro show that the entrant may choose to enter earlier than is socially optimal.

II. CONSUMER PREFERENCES AND BEHAVIOR

We suppose that consumers live for two periods and purchase only when young.³ There is a constant mass of new consumers each period that we normalize to equal 1, so there is total mass 2 of consumers at each date. We assume there are two types of consumers: A proportion λ of the consumers are ‘high types’ with value θ_h , and proportion $1 - \lambda$ are ‘low types’ with value θ_l , $\theta_h > \theta_l > 0$.⁴

The per-period utility of consumer θ from the good when the network has size x is $\theta + vx + \gamma t$, where γ is the rate of technological progress, v measures the importance of network externalities, and both of these parameters are positive. Potential purchasers (i.e. new consumers) have access to an outside option that is also improving over time. For simplicity, we assume that the outside option is not a network good; its value at date t is $\gamma(t-1)$.⁵

Note that the assumption that consumers only buy when they are young also applies to the outside option. Thus the net benefit of the good over the outside option is $\theta + vx + \gamma$.⁶

³This assumption lets us avoid issues of ‘Coasian dynamics’ (e.g. lower prices in the second period of a consumer’s life) and also of the pricing of upgrades. See Fudenberg and Tirole [1998] for a general discussion of upgrade pricing and Ellison and Fudenberg [1999] for an analysis of upgrades of network goods.

⁴The assumption of two types and two-period lives will substantially simplify the dynamics, for the transition to the steady-state output level from any initial condition will always occur in a single period. We conjecture that with a continuum of types, the adjustment paths would be gradual instead of immediate; see the remark after the statement of Theorem 2.

⁵Section V discusses relaxing this assumption.

⁶The assumption of linear network externalities is convenient, and it is standard in the literature. However, the only explanations we know of for the linear form are for cases where the network externality is ‘direct’, as with the size of a phone network. If the externality is indirect, through increasing returns in developing applications, then we do not know how to justify the linear form. (See Nahm [1999] for an alternative.) Also note that our specification of preferences supposes that the effect of network externalities is the same for all consumers, which means that a monopoly supplier can extract all of the benefits that consumers get from the network externality. Ellison and Fudenberg [1999] argue that it is more natural to suppose that people who value the good more highly also place more value on its network benefits, in this case the high types retain some rents from the network externality. Likewise, we assume here that all consumers have the same value for innovation, so that this benefit is also fully appropriated by the firms. As we will see below, this assumption leans in the direction of finding there is ‘too much’ incentive for entry.

Consumers and the firm have the same discount factor δ . Thus consumer θ at date t is willing to pay $(1 + \delta)(\theta + \gamma) + \nu x_t + \delta \nu x_{t+1}$ to purchase the good, where x_t, x_{t+1} are the forecasted sizes of the good's network at dates t and $t + 1$. (Note that in a steady state with all customers purchasing, $x_t = x_{t+1} = 2$, while in a steady state with only 'high' value customers purchasing, $x_t = x_{t+1} = 2\lambda < 2$.)

Before considering how prices are determined, we need to compute the demand curve, i.e. the set of consumers who will purchase at each possible price. This step is complicated in models of network goods because there can be multiple equilibrium responses to a given price. For example, in static network models with identical consumers there are typically some prices for which both 'everyone purchase' and 'no one purchase' are equilibria. This occurs when the price is low enough that is worth buying if everyone buys, but the price is above the value derived from being the only user of the good. To handle the multiple equilibria, we will follow convention and suppose that once a fixed price is announced consumers coordinate on the equilibrium continuation that they prefer, which we will show to be the one with the highest volume of sales.⁷ Our initial results will not depend on this assumption; we will make it explicit at the point where it is first used.

III. PRICING BY AN INCUMBENT WITHOUT THE THREAT OF ENTRY

Our goal in this section is to determine how prices are set when the threat of entry is absent. Because the willingness to pay of current consumers depends on both the 'installed base' of consumers who purchased in the previous period and on expectations about next period's sales, this situation corresponds to a non-trivial dynamic game, and its resolution may be of some independent interest. However, our goal here is not a complete characterization of the outcome for all parameters, but rather to establish that for some parameters the unique equilibrium outcome is for the incumbent to sell only to the high-value types. We will then proceed in the next section to study whether the threat of entry will lead the incumbent to lower its prices and sell to all consumers.

We will restrict attention to 'Markov-perfect equilibria' (MPE) in which strategies depend on the history only through the payoff-relevant state variables.⁸ In this model there are two sub-periods at each date t : first the firm chooses a price p_t , and then the consumers decide whether or not to purchase. When the firm is choosing p_t , the only payoff-relevant variable at date t is the size of the 'installed base' q_{t-1} of consumers who purchased

⁷ This is the equilibrium selection used by Farrell and Saloner [1985] and Katz and Shapiro [1986].

⁸ See Maskin and Tirole [1998]. This concept has also been called 'state-space equilibrium'.

in the previous period, so the restriction to Markov perfection here means that the firm's strategy can be expressed a function of q_{t-1} . That is, at any date t , the evolution of prices and output from date $t+1$ on depends on the previous period's sales q_{t-1} (and other aspects of past play) only through the effect of q_{t-1} on the current sales, q_t .

Thus, in a pure-strategy equilibrium we can write $q_t = Q^*(q_{t-1})$, and let $V(q_{t-1})$ be the incumbent's equilibrium continuation value from period t on when sales at $t-1$ were q_{t-1} . When consumers are making their decisions, the state comprises both the installed base q_{t-1} and the posted price p_t . Here Markov-perfection requires that the active consumers behave the same at any two histories for which their future flow of payoffs is the same under any sequence of future actions, so that the behavior of consumers at date t is determined by the value of $p_t - vq_{t-1}$, which is the 'net price' of the good after allowing for the benefit provided by the installed base.

We suppose that production is costless, so that the incumbent's goal is to maximize the present value of its revenue. Some insight into the MPE outcome can be obtained by comparing the incumbent's revenue stream across steady states, where the consumers are assumed to have correct expectations. If the incumbent sells to all consumers every period, then consumers receive network benefits of $2v$ each period, and the incumbent's steady state revenue is $(1 + \delta)(\theta_l + 2v + \gamma)$, which we define to be $\pi^*(1)$. If the incumbent sells only to the high value types, network benefits are $2\lambda v$ each period, and so the incumbent's steady state revenue is $(1 + \delta)\lambda(\theta_h + 2\lambda v + \gamma) \equiv \pi^*(\lambda)$.

Thus $\pi^*(\lambda) > \pi^*(1)$ iff

$$(A1) \quad \lambda\theta_h - \theta_l > (1 - \lambda)\gamma + 2v(1 - \lambda^2),$$

which is what we will assume. (Note that A1 is satisfied whenever the proportion λ of high types is sufficiently large.)

Our goal is to prove that under A1, in any pure-strategy MPE the incumbent sells only to high types. As a first step we show that this is *an* MPE.

Lemma 1. Under A1, the following is an MPE:

- Low-type consumers purchase at date t iff $p_t \leq \underline{p} + vq_{t-1}$, where

$$\underline{p} = (1 + \delta)(\theta_l + \gamma + v) + v\delta\lambda$$

- High-type consumers purchase iff $p_t \leq \bar{p} + vq_{t-1}$, where

$$\bar{p} = (1 + \delta)(\theta_h + \gamma + v\lambda) + v\delta\lambda$$

- The firm's strategy is to set $p(q_{t-1}) = \bar{p} + vq_{t-1}$ for all q_{t-1} , so $Q^*(q_{t-1}) = \lambda$ for all q_{t-1} .

Remark. Note that $\bar{p} - \underline{p} > 0$ from A1, so there is an interval of prices such that only the high types buy. In this interval, purchasing is not individually rational for the low types even if all consumers purchase, but it is individually rational for high types to purchase if only high types purchase.

Proof. Given the incumbent's strategy, the consumers' strategy is clearly an equilibrium. (In fact at each date t and price p_t this is the best equilibrium in the coordination game between current consumers, holding the incumbent's pricing policy fixed.)

Given the consumers' strategy, the incumbent's value maximizing choice will either be $\bar{p} + vq_{t-1}$ or $\underline{p} + vq_{t-1}$; and the higher price is preferred *iff*

$$(1) \quad \lambda[\bar{p} + vq_{t-1}] + \delta V(\lambda) \geq \underline{p} + vq_{t-1} + \delta V(1).$$

Thus (1) holds for all q_{t-1} *iff*

$$(2) \quad \lambda[\bar{p} + v] - \underline{p} - v \geq \delta[V(1) - V(\lambda)].$$

Given the incumbent's strategy, the only future difference between $q_t = 1$ and $q_t = \lambda$ is that the price at $(t+1)$ increases by $v(1 - \lambda)$, so using the one-stage-deviation principle $V(1) - V(\lambda) = v\lambda(1 - \lambda)$. Thus (2) simplifies to

$$(3) \quad \lambda\bar{p} - \underline{p} \geq v(1 - \lambda)(1 + \delta\lambda)$$

and substituting for the cut-off prices \bar{p}, \underline{p} we have

$$(4) \quad \lambda[(1 + \delta)(\theta_h + \gamma + v\lambda) + v\delta\lambda] - [(1 + \delta)(\theta_l + \gamma + v) + v\delta\lambda] \geq v(1 - \lambda)(1 + \delta\lambda), \text{ or}$$

$$(5) \quad \lambda\theta_h - \theta_l \geq \frac{(1 - \lambda)}{(1 + \delta)}[(1 + \delta)[\gamma + (1 + \lambda)v] + 2\delta v\lambda + v] = [(1 - \lambda)\gamma + (1 - \lambda^2)2v] - \frac{(1 - \lambda)v(\delta + \lambda - \delta\lambda)}{1 + \delta}.$$

Inequality (5) is implied by A1, which says that $\lambda\theta_h - \theta_l > \gamma(1 - \lambda) + 2v(1 - \lambda^2)$. Q.E.D.

Remark. The proof above actually shows that there is an MPE with output λ even for some parameters where steady state profit is higher when selling to all customers. The reason is that in this MPE the incumbent takes the consumer's beliefs about next period's output to be fixed at $q_{t+1} \equiv \lambda$, while in the commitment case the incumbent can commit to $q_{t+1} = 1$. This matters, because if customers expect next period's output to be low, they have a lower value for the good, which can make it optimal for the incumbent to restrict output.

Lemma 2. In a pure-strategy MPE, the value function $V(q_t)$ is a non-
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decreasing continuous function of q_t , and output $q_t = Q^*(q_{t-1})$ is a non-decreasing function of the installed base q_{t-1} .

Proof. See Appendix. A revealed-preference-like argument shows that V is continuous and non-decreasing. The result that the incumbent's chosen output is a non-decreasing function of the installed base rests on the following intuition: When the base grows, a price increase of exactly the increase in network benefit must lead to the same sales as before (from the MPE assumption.) Because the price is now higher, it becomes more costly to restrict sales, so if output changes at all it must increase.

Given the monotonicity established by lemma 2, when there are multiple MPE continuations in the subperiod at date t after p_t is announced, date t consumers always at least weakly prefer the continuation with the highest current sales. Henceforth we will assume that consumers coordinate in this way, so that the only possible equilibrium output levels are $q = 0$, $q = \lambda$ and $q = 1$. We will call this the 'coordination assumption'; it is the dynamic analog of the standard equilibrium selection in static network models.

Theorem 1. Under A1 and the coordination assumption, the unique pure-strategy equilibrium is to always sell to only the high-value consumers, i.e. $Q^*(q_{t-1}) = \lambda$ for all q_{t-1} .

Proof. Fix a pure strategy MPE. The highest price consistent with all low-type consumers purchasing when $q_{t-1} = 0$ is

$$(6) \quad \underline{p}' = (1 + \delta)(\theta_l + \gamma + v) + v\delta Q^*(1),$$

and the highest price consistent with all high types purchasing when $q_{t-1} = 0$ is

$$(7) \quad \bar{p}' = (1 + \delta)(\theta_h + \gamma + v\lambda) + v\delta Q^*(\lambda).$$

For general q_{t-1} these prices become $\underline{p}' + vq_{t-1} \equiv p_l(q_{t-1})$ and $\bar{p}' + vq_{t-1} \equiv p_h(q_{t-1})$ respectively.

Since the incumbent's optimum is to charge the highest price consistent with a given level of sales, we have that $V(q_{t-1}) = \max\{\lambda p_h(q_{t-1}) + \delta V(\lambda), p_l(q_{t-1}) + \delta V(1)\}$.

Thus our claim that the incumbent always charges the high price is equivalent to

$$(8) \quad \lambda p_h(q_{t-1}) + \delta V(\lambda) > p_l(q_{t-1}) + \delta V(1) \text{ for all } q_{t-1}.$$

Because our model has only two types, monotonicity implies there are only two possible MPE other than constant low output: Either output is

increasing in the installed base, so $Q^*(1) = 1$ and $Q^*(\lambda) = \lambda$, or output is always high, $Q^*(\lambda) = Q^*(1) = Q^*$.

In the first case, condition (8) becomes

$$\frac{\pi^*(\lambda)}{1-\delta} + \lambda v(q_{t-1} - \lambda) > \frac{\pi^*(1)}{1-\delta} + v(q_{t-1} - 1), \text{ or}$$

$\pi^*(\lambda) - \pi^*(1) > (1-\lambda)(1-\delta)v[q_{t-1} - (1+\lambda)]$; since the final right-hand side expression is negative, this follows from A1.⁹

In the second case, where $Q^*(\lambda) = Q^*(1) = Q^*$, then $V(1) - V(\lambda) = v(1-\lambda)Q^*$, and substituting for p_l, p_h condition (8) reduces to¹⁰

$$(9) \quad (1+\delta)[\lambda\theta_h - \theta_l - \gamma(1-\lambda) + v(\lambda^2 - 1)] > (1-\lambda)v[2\delta Q^* - q_{t-1}]$$

From (A1) $\lambda\theta_h - \theta_l - \gamma(1-\lambda) + v(\lambda^2 - 1) > v(1-\lambda^2)$, so a sufficient condition for (9) is

$$(10) \quad (1+\delta)[1-\lambda^2] > (1-\lambda)[2\delta Q^* - q_{t-1}],$$

which is true for all q_{t-1}, Q^*, δ and λ between 0 and 1. Q.E.D.

IV. ENTRANTS AND ENTRY DETERRENCE

Each period there is a new potential entrant with an entry cost k . The entry costs of each entrant are random, and are independently and identically distributed according to distribution F on $[0, \infty)$. The date- t entrant learns its cost at the beginning of date t . It then decides whether or not to enter; if it chooses not to enter its payoff is zero and it leaves the game. In particular, the date- t entrant is only able to enter in period t ; it is not able to wait and enter at $(t+1)$. If it enters, the entrant provides a better and *incompatible* product. The entrant's date- t technology is assumed to be better than the incumbent's by an amount γ_0 , so that it gives extra utility (at the same level of network benefits) of $(1+\delta)\gamma_0$ to date- t consumers. Moreover, successful entry at period t is assumed to completely 'spillover' at all subsequent dates, giving rise to a sequence $\gamma_1, \gamma_2, \dots$ of improvements to the technology of the successful entrant, future potential entrants, and the outside option in periods $t+1, t+2, \dots$. This assumption of complete

⁹ The displayed equation follows from

$$\begin{aligned} \pi^*(\lambda) - \pi^*(1) &> (1-\delta)v[(q_{t-1} - 1) - \lambda(q_{t-1} - \lambda)] = (1-\delta)v[(1-\lambda)q_{t-1} - (1-\lambda^2)] \\ &= (1-\lambda)(1-\delta)v[q_{t-1} - (1+\lambda)]. \end{aligned}$$

¹⁰ To derive (9), first substitute for p_l, p_h in (8) yielding

$$\lambda[(1+\delta)(\theta_h + \gamma + v\lambda) + v\delta Q^*(\lambda) + vq_{t-1}] + \delta V(\lambda) > (1+\delta)(\theta_l + \gamma + v) + v\delta Q^*(1) + vq_{t-1} + \delta V(1)$$

or

$$(1+\delta)[\lambda\theta_h - \theta_l - \gamma(1-\lambda) + v(\lambda^2 - 1)] > v[\delta Q^*(1) + (1-\lambda)q_{t-1} - \lambda\delta Q^*(\lambda)] + \delta(V(1) - V(\lambda)).$$

Then substitute $V(1) - V(\lambda) = v(1-\lambda)Q^*$.

spillovers, together with the assumption that the total size of the market is fixed, implies that the competitive environment of an entrant who successfully takes over the market is identical to the one that the incumbent would have faced, so that we do not need to track the dates that entry has occurred, or even the number of successful entries, when analyzing the competitive environment.¹¹

We assume that the technological leap caused by entry is ‘large’ compared to the network effects in the following sense:

$$(A2) \quad (1 + \delta)\gamma_0 > v.$$

This means that the entrant’s technological edge outweighs even the largest possible installed base advantage of the incumbent.

We look at ‘limit pricing equilibria’ in which:

- Each entrant enters *iff* its cost is less than a critical value k^* .
- If the entrant does come in at date t , it captures the entire market in that period, the incumbent leaves at $t + 1$, and the entrant becomes the new incumbent.
- Incumbents choose to sell to all consumers rather than just the high-value ones in order to reduce the probability of entry in the future. That is, in any period where entry does not occur the incumbent’s sales are equal to 1.

Note that the choice of which firm to buy from at given prices again has a coordination aspect and has multiple equilibria—if the two firms’ quality-adjusted prices are sufficiently close, each consumer wants to buy from whichever firm he thinks everyone else is buying from. As above, we make the standard assumption that, given prices (p_I, p_E) for the incumbent and entrant respectively, consumers at date t coordinate on the equilibrium that is best *for them*, ignoring the utility of the previous period’s consumers (who would rather everyone stayed with the old good.) In making this decision, consumers realize that if they all buy from the entrant, the incumbent will leave the market, and that conversely if they all buy from the incumbent the entrant will leave.¹² Because next period the two firms will have the same technology, the choice of which firm to coordinate on now has no effect on next period’s network externalities, so the choice of whether to buy from the incumbent or the entrant depends only on (1) the prices, (2) the incumbent’s installed based inherited from date $t - 1$, and (3) the utility gain of $(1 + \delta)\gamma_0$ that come from the entrant’s

¹¹ Of course, the number of successful entries does have an impact on welfare, as the technological spillovers are appropriated by the consumers. The present value of the spillover from each successful entry is $S = (1 + \delta) \sum_{n=1}^{\infty} \delta^n \gamma_n$.

¹² This forecast is based on the fact that next period the two firms will have the same technology, so that coordination by the next generation will ensure that the firm with the larger installed base gets all of the sales.

good being better by γ_0 . Thus all consumers will prefer the entrant to the incumbent if

$$(12) \quad p_E - p_I \leq (1 + \delta)\gamma_0 - [v(q_{t-1} + 1) - v(1)];$$

(A2) ensures that the right-hand side of inequality (12) is positive.

We will consider two cases, with and without the constraint that prices be non-negative. Because we have normalized cost to equal 0, prices in our model correspond to mark-ups; in a more general model the issue is whether markups can be so negative and larger in absolute value than unit cost. The two cases are similar in many respects so we begin with the simpler case of unrestricted prices.

Unrestricted Prices

With unrestricted prices, Bertrand competition results in ‘complete rent dissipation’ in the period where entry occurs, in the sense that it drives the incumbent’s price down to the point where setting a lower price today and driving off the current entrant would have a negative NPV. Formally, let $V(q_{t-1})$ be the present value of being the incumbent at the start of period t when sales at $t-1$ were q_{t-1} . Then we assume that when entry occurs the incumbent sets price $p_I = -\delta V(1)$. The entrant then prices just low enough that consumers prefer to purchase from it, that is $p_E(q_{t-1}) = (1 + \delta)\gamma_0 - vq_{t-1} - \delta V(1)$; this ‘introductory offer’ results in all consumers buying from the entrant. It also results in the entrant ‘rebating’ to consumers the discounted value of its future profits, so that the payoff gross of entry costs is simply the entrant’s efficiency advantage of $(1 + \delta)\gamma_0 - vq_{t-1}$. Hence the equilibrium value of the entry threshold is

$$(13) \quad k^*(q_{t-1}) = (1 + \delta)\gamma_0 - vq_{t-1}.$$

Let the equilibrium probability of entry be $y(q_{t-1}) = F(k^*(q_{t-1}))$. Then in an equilibrium where the incumbent’s equilibrium strategy is to sell to all consumers regardless of the history, the willingness to pay of type θ when the installed base is q_{t-1} is $(1 + \delta)(\gamma + \theta) + v((1 + q_{t-1}) + \delta(2 - y(1)))$. Here the term $v(1 + q_{t-1})$ measures network benefits in the current period, and the term $v\delta(2 - y(1))$ measures the expected value of network benefits at date $t + 1$, since if entry occurs at date $t + 1$ the incumbent’s sales will be 0, and because the entrant’s good is incompatible the date- $t + 1$ network benefit from the incumbent’s good will be v instead of $2v$.

Since the incumbent’s value is 0 if entry occurs, the incumbent’s equilibrium value at the beginning of period t , before the current entrant’s cost is realized, is

$$(14) \quad V(q_{t-1}) = [1 - y(q_{t-1})][(1 + \delta)(\gamma + \theta_l) + v((1 + q_{t-1}) + \delta(2 - y(1))) + \delta V(1)].$$

Note finally that the equilibrium value is

$$V(1) = \frac{[1 - y(1)][((1 + \delta)(\gamma + \theta_l)) + v(2(1 + \delta) - \delta y(1))]}{1 - \delta(1 - y(1))}.$$

We want to show that it is an equilibrium for the incumbent to set $q_t = 1$ for any value of q_{t-1} when entry does not occur. Following the lines of lemma 2, we can show that the incumbent's output is weakly monotone in q_{t-1} , so it is sufficient that this be true for $q_{t-1} = 0$, i.e. that

$$(15) \quad \begin{aligned} & (1 + \delta)(\gamma + \theta_l) + v(1 + \delta(2 - y(1))) + \delta V(1) \\ & \geq \lambda[(1 + \delta)(\gamma + \theta_h) + v(\lambda + \delta(1 + \lambda - y(\lambda))) + \delta V(\lambda)]. \end{aligned}$$

Theorem 2. If A2, (13), (14), and (15) are satisfied, and firms can set negative prices, there is a limit-pricing equilibrium in which the current incumbent always produces high output. Along the equilibrium path, each potential entrant enters iff its realized cost is less than $k^*(1) = (1 + \delta)\gamma_0 - v$. Moreover, these conditions are compatible with condition A1, under which the incumbent sets low output when entry is impossible.

Remark. The reason parameter restrictions are needed in Theorem 2 is that we are considering a two-type model where the only form of limit pricing is to sell to all consumers. In a model with a continuum of types, we would expect that an incumbent faced with potential entry would always engage in limit pricing in the weaker sense of setting a higher output and lower price than if entry were impossible. We have not developed a continuum-of-types model because its dynamics are more complicated: in such a model, the adjustment path from high sales in an entry period to the steady-state output would be gradual instead of immediate, and indeed the steady state might only be approached asymptotically.

We next explore the hypotheses of Theorem 2 in more detail, and present some examples where they are all satisfied. Since a very impatient firm would not be willing to forego current profits to deter future entry, we only expect limit pricing to occur for discount factors that are sufficiently large. For analytic simplicity, our analysis will focus on the polar opposite case of discount factors near to 1.¹³

¹³Remember that the period length here corresponds to the minimum time between successive generations of the product. When this period is very short (which is one reason that the discount factor might be close to 1) it seems plausible that consumers might remain active for more than two periods. We briefly discuss this extension of the model in the concluding section.

Substituting (14) into (15) and rearranging terms shows (see appendix) that a sufficient condition for (15) is

$$(16) \quad \delta(y(\lambda) - y(1)) \left[\frac{V(1)}{1 - y(1)} + v \right] - 2\delta v(1 - \lambda)y(\lambda) \\ \geq (1 + \delta)(\lambda\theta_h - ((1 - \lambda)\gamma + \theta_l) + v(\lambda^2 - 1)) + 2\delta v(\lambda - 1).$$

Whether this condition is satisfied depends on the discount factor of the incumbent, and the extent to which increased output decreases entry, that is $y(\lambda) - y(1)$, which in turn depends on the distribution of the entrant's costs. The simplest, albeit somewhat artificial, example where limit pricing occurs is when costs have support $[k^*(1), k^*(\lambda)] = [(1 + \delta)\gamma_0 - v, (1 + \delta)\gamma_0 - v\lambda]$.

Here $y^*(1) = 0$ and $y^*(\lambda) = 1$, so $V(1) = \frac{(1 + \delta)(\gamma + \theta_l + 2v)}{1 - \delta}$; since $V(1)$ becomes arbitrarily large as $\delta \rightarrow 1$ it is clear that there are a range of discount factors close to 1 where (16) is satisfied.

More generally, algebraic manipulations show that (16) will be satisfied, and a limit-pricing equilibrium will exist for discount factors close to 1, if the limiting value of $\frac{y(\lambda) - y(1)}{y(1)}$ is sufficiently large, while the case we considered above had this ratio tending to infinity.¹⁴ In words, the key is that along the proposed equilibrium path of $q_t = 1$ for all t , deviating to the short-run monopoly output leads to a sufficiently large percentage increase in the probability of entry; this in turn depends on the details of the consumer's utility function and the nature of the distribution F of entry costs. The working paper version of this paper presents a specific example, in which $\lambda = \gamma = v = \gamma_0$, and entry costs are uniformly distributed on $[0, 2v]$, independent of δ , and shows that the sufficient conditions for the limit pricing equilibrium are consistent with A1 and A2.

Welfare Implications of Limit Pricing

Along the equilibrium path, entry at date t has the following effects on welfare:

¹⁴ To see this, note that

$$\lim_{\delta \rightarrow 1} V(1) = \frac{[1 - y(1)][(2(\gamma + \theta_l) + v(4 - y(1)))]}{y(1)}.$$

Thus as $\delta \rightarrow 1$, (16) reduces to

$$(y(\lambda) - y(1)) \left[\frac{(2(\gamma + \theta_l) + v(4 - y(1)))}{y(1)} + v \right] - 2v(1 - \lambda)y(\lambda) \\ \geq 2(\lambda\theta_h - ((1 - \lambda)\gamma + \theta_l) + v(\lambda^2 - 1)) + 2v(\lambda - 1).$$

- (A) Date- t consumers get a higher-quality good and less network benefits; the increased consumption utility $(1 + \delta)\gamma_0 - v$.
- (B) The entrant incurs an entry cost of k .
- (C) Date- $(t - 1)$ consumers get network benefits v instead of $2v$.
- (D) Consumers from $(t + 1)$ on gain $S = (1 + \delta) \sum_{n=1}^{\infty} \delta^n \gamma_n$.

The entrant's value gross of entry cost is exactly equal to the first term, and the entrant pays the entry cost itself. Thus the entrant perfectly internalizes effects (A) and (B), so when it has cost $k = k^*(1)$ and is just indifferent about entry, these first two effects cancel each other out. However, the entrant considers neither the negative externality that entry imposes on old consumers nor the benefit accruing from technological spillovers.¹⁵

To see this algebraically, compute the welfare effect of entry by summing terms (A)–(D): $\Delta W = (1 + \delta)\gamma_0 - 2v - k + S$.

Since the entry threshold $k^*(1) = (1 + \delta)\gamma_0 - v$ is at the point where the private gains from entry just equal the cost, the welfare impact of entry by an entrant whose cost is just at the threshold is the difference between the social and private return to entry, which is $-v + S$. Hence in the absence of technological spillovers, welfare would be increased by a government policy that discouraged entry and so increased k^* , while in general the welfare comparison depends on the comparison of network benefits to the present value of the spillover.

Non-Negative Prices/Markups

So far we have assumed that the firms can set negative prices. If instead we restrict prices to be non-negative, then competition does not completely dissipate the firms' rents. Instead, when entry occurs the incumbent sets price equal to 0, and the date- t entrant sets price equal to its net advantage, which is $(1 + \delta)\gamma_0 - vq_{t-1}$. Thus the net payoff from entry is $(1 + \delta)\gamma_0 - vq_{t-1} + \delta V(1) - k$, so the entrants set

$$(17) \quad k^*(q_{t-1}) = (1 + \delta)\gamma - vq_{t-1} + \delta V(1).$$

Replacing (13) with (17) implies higher values for $y(1)$ and $y(\lambda)$ than above, but with the new values equations (14) and (15) continue to be sufficient for the existence of a limit pricing equilibrium, and these conditions can still be satisfied at the same time as A1. The main difference is in the comparison between equilibrium and socially optimal entry: While the welfare effects of entry are the same as before, the entrant's private return

¹⁵With a more general distribution of consumers we expect that sales might be higher in the entry period than in the steady state, which would add an extra welfare benefit to entry.

to entry is now larger by the term $\delta V(1)$ which represents the entrant's reward from 'stealing business' from the incumbent. Because of this 'business stealing' effect, which has no welfare counterpart, the welfare impact of entry by the marginal entrant is now $-v + S - \delta V(1)$, so entry is more likely to be excessive than under unrestricted prices.

V. EXTENSIONS

Limit Pricing and the Size of Innovation

So far we have supposed that the entrants use a common and fixed process for generating new technologies and hence entry opportunities. In this section we examine a simple model where entrants can choose one of two research processes; for simplicity we use the model where prices are allowed to be negative.

Specifically, we suppose that entrants have a choice between technology 1, with improvement γ_0 over the current state of the art, and costs k distributed according to $F(k)$, and technology 2, which has improvement $\hat{\gamma}_0 > \gamma_0$ and has costs \hat{k} distributed according to some $\hat{F}(k)$.¹⁶ We assume moreover that $\lambda v < (1 + \delta)\gamma_0 < v < (1 + \delta)\hat{\gamma}_0$; this and our selection of the consumers' preferred equilibrium implies that when $q_{t-1} = 1$, and the entrant selects technology 1, consumers would prefer to buy from the incumbent even if the entrant sets its price so low as to dissipate all future returns, but that entry would be successful with technology 1 when $q_{t-1} = \lambda$ and that entry is always successful with technology 2. Hence when $q_{t-1} = 1$ the entrant will choose technology 2, while if $q_{t-1} = \lambda$ the entrant will compare $\Pr[k \leq (1 + \delta)\gamma_0 - v\lambda] [(1 + \delta)\gamma_0 - v\lambda - E(k|k \leq (1 + \delta)\gamma_0 - v\lambda)]$ and $\Pr[\hat{k} \leq (1 + \delta)\hat{\gamma}_0 - v\lambda] [(1 + \delta)\hat{\gamma}_0 - v\lambda - E(\hat{k}|\hat{k} \leq (1 + \delta)\hat{\gamma}_0 - v\lambda)]$.

Since the entrant only captures one period of the increased utility from innovation, but not the future value of spillovers, it is easy to construct cases where the privately optimal choice when $q_{t-1} = \lambda$ is technology 1, even though choosing technology 2 would lead to higher social welfare. Hence in addition to the static benefit of higher output and lower prices, limit pricing can increase welfare by encouraging potential entrants to aim for riskier but more substantial improvements over current technology instead of safer, incremental ones. Of course in other models limit pricing may lead entrants to make technology choices that lower welfare. Since the size of this and the other effects described in this paper is an empirical question, our goal here is to identify some of the relevant empirical issues. We hope that they will be explored in future work.

¹⁶In order for the technology choice to be non-trivial, the cost of technology 2 must tend to be higher than that of technology 1.

Different Specifications of Market Demand

We have already commented on the complications and likely effects of relaxing the assumption that there are only two types of consumer; we now comment briefly on several other alternative specifications of market demand.

First of all, consider the effect of allowing the outside option to be a network good. In the monopoly model, the monopolist has less incentive to raise prices, because lowering its sales increases the network benefit of the outside option. Thus condition A1, which says that steady-state profits are higher when selling to only the high-value types, would need to be modified. But we conjecture that Lemma 1 and Theorem 1 would extend with this modified version of the condition: selling to the high types would be a MPE, and the unique MPE that meets the coordination assumption. In the entry model, the current availability of the outside option would not matter in periods where entry occurs, while its effect in periods without entry would be very similar to that in the monopoly model. Thus we conjecture that the limit-pricing equilibrium would look very similar, although the conditions for its existence would be changed, and once again we would have a quantitative change but not a qualitative one.

Next, suppose that consumers remain active for three or more periods instead of two. Allowing new consumers to delay purchasing, or older consumers to upgrade to new versions, introduces the issues of Coasian dynamics and upgrade pricing, but these complications can be avoided if we maintain the assumption that consumers can purchase only in the first period that they are active. In this case, the installed base forms a larger portion of the total market, so the entrant's initial disadvantage is larger, and so one might expect there to be less entry for a given distribution of entry costs.¹⁷ However, there are several complications that make it hard to tell if these intuitions are correct without fully analyzing the model. First of all, it is not obvious that one wants to hold fixed the distribution of entry costs when comparing two models with different lifetimes or discount factors. Second, once consumers are active for more than two periods, one must consider 'entry paths' along which the incumbent remains active for several periods after entry occurs.¹⁸ Third, both consumers and firms need to consider the possibility that at some times there will be more than two active and incompatible networks. These complications seem interesting, and could be worth analyzing fully.

¹⁷ Note though a reduction in entry increases the value of being an incumbent, which increases the returns to entry, so the size of this effect is not clear.

¹⁸ Put differently, if the entrant assumed that the incumbent was inactive and charged the steady-state limit price then consumers might prefer to purchase from the incumbent, so the incumbent's post-entry prices cannot be ignored. This issue was already noted in Katz and Shapiro [1992].

Other issues may arise if demand is non-stationary. For example, suppose that as in our model consumers are only active for 2 periods, but that the mass of new consumers grows at rate a for the first T periods before leveling off. Since the installed base is relatively less important when demand is growing, entry is easier at the start of the model, and equilibrium limit pricing might be aggressive early on and closer to monopoly pricing when the market is mature. This is another model that might merit further study.

APPENDIX

Proof of Lemma 2

Fix an arbitrary pure strategy MPE, and let the equilibrium price be $P^*(q_{t-1})$. At any state q_{t-1} the incumbent has the option of setting $p_t = P^*(q'_{t-1}) + vq_{t-1} - vq'_{t-1}$ for any other $q'_{t-1} \in [0, 1]$; this yields sales $Q^*(q'_{t-1})$ and an overall payoff of $V(q'_{t-1}) + v(q_{t-1} - q'_{t-1})Q^*(q'_{t-1})$. Since sales are between 0 and 1 this shows that V is continuous and non-decreasing.

Next, note that from the MPE assumption, the evolution of play from period $t+1$ on is completely determined by q_t , so if there are two or more prices that yield exactly the same level of current sales the incumbent will chose the highest of them. Moreover, since the value function is increasing, the incumbent will not set price p if some price $p' > p$ yields higher sales. Let $x_t = X^*(p - vq_{t-1})$ denote the amount of sales at a given pair (p, q_{t-1}) . We have not shown that the 'demand function' X^* is continuous, so let $\hat{p}(q|q_{t-1}) = \sup\{p|X^*(p - vq_{t-1}) = q\}$; the MPE assumption implies that $\hat{p}(q|q_{t-1}) = \hat{p}(q|0) + vq_{t-1}$.

Since the price at date t is chosen to solve $\max_p [pX^*(p - vq_{t-1}) + \delta V(X^*(p - vq_{t-1}))]$, and V is continuous, in any equilibrium the supremum is attained by the equilibrium price.¹⁹ Thus we can think of the incumbent selecting period- t output q_t instead of the price, with the price given by $\hat{p}(q_t|q_{t-1})$. Hence

$$\begin{aligned} V(q_{t-1}) &= \max_p [pX^*(p - vq_{t-1}) + \delta V(X^*(p - vq_{t-1}))] \\ &= \max_p [q\hat{p}(q|q_{t-1}) + \delta V(q)]. \end{aligned}$$

Fix a pair of states q_{t-1}, q'_{t-1} and let $q = Q^*(q_{t-1})$, $q' = Q^*(q'_{t-1})$. A standard revealed preference argument now shows that

$$q'[\hat{p}(q'|q'_{t-1}) - \hat{p}(q'|q_{t-1})] \geq q[\hat{p}(q|q'_{t-1}) - \hat{p}(q|q_{t-1})], \text{ or } (q' - q)(q'_{t-1} - q_{t-1}) \geq 0.$$

Q.E.D.

Proof that inequality (16) is sufficient for inequality (15)

First rewrite (15) as

$$\begin{aligned} \delta(V(1) - V(\lambda)) \\ \geq (1 + \delta)(\lambda\theta_h - ((1 - \lambda)\gamma + \theta_l) + v(\lambda^2 - 1)) + \delta v(\lambda - 1) + v\delta(y(1) - \lambda y(\lambda)). \end{aligned}$$

¹⁹ Because we are analyzing an MPE, we know that the incumbent's maximization problem has a solution for each q_{t-1} .

From (14), $V(1) - V(\lambda) = \frac{y(\lambda) - y(1)}{1 - y(1)} V(1) + v(1 - \lambda)(1 - y(\lambda))$; substituting this in to (15) yields

$$\begin{aligned} & \delta(y(\lambda) - y(1)) \left[\frac{V(1)}{1 - y(1)} \right] + \delta v(1 - \lambda)(1 - y(\lambda)) \\ & \geq (1 + \delta)(\lambda\theta_h - ((1 - \lambda)\gamma + \theta_l) + v(\lambda^2 - 1)) + \delta v(\lambda - 1) + v\delta(y(1) - \lambda y(\lambda)) \end{aligned}$$

Rearranging terms gives

$$\begin{aligned} & \delta(y(\lambda) - y(1)) \left[\frac{V(1)}{1 - y(1)} \right] - \delta v(1 - \lambda)y(\lambda) \\ & \geq (1 + \delta)(\lambda\theta_h - ((1 - \lambda)\gamma + \theta_l) + v(\lambda^2 - 1)) + 2\delta v(\lambda - 1) + v\delta(y(1) - \lambda y(\lambda)) \\ & = (1 + \delta)((\lambda\theta_h - ((1 - \lambda)\gamma + \theta_l) + v(\lambda^2 - 1)) + 2\delta v(\lambda - 1) + v\delta(y(1) - y(\lambda)) \\ & \quad + v\delta(1 - \lambda)y(\lambda)). \end{aligned}$$

A sufficient condition for this is

$$\begin{aligned} & \delta(y(\lambda) - y(1)) \left[\frac{V(1)}{1 - y(1)} + v \right] - 2\delta v(1 - \lambda)y(\lambda) \\ & \geq (1 + \delta)(\lambda\theta_h - ((1 - \lambda)\gamma + \theta_l) + v(\lambda^2 - 1)) + 2\delta v(\lambda - 1) \end{aligned}$$

which is (16).

REFERENCES

Baumol, J., Panzar, J. and Willig, R., 1982, *Contestable Markets and the Theory of Industry* (Harcourt, Brace, and Jovanovich, NY).

Eaton, B. C. and Lipsey, R., 1980, 'Exit Barriers are Entry Barriers: The Durability of Capital as a Barrier to Entry', *Bell Journal of Economics*, 12, pp. 721-729.

Ellison, G. and Fudenberg, D., 1999, 'The Neo-Luddite's Lament: Too Many Upgrades in the Software Industry', forthcoming in the *Rand Journal of Economics*.

Farrell, J. and Saloner, G., 1985, 'Standardization, Compatibility, and Innovation', *Rand Journal of Economics*, 16, pp. 70-83.

Farrell, J. and Saloner, G., 1986, 'Installed Base and Compatibility', *American Economic Review*, 76, pp. 940-955.

Fudenberg, D. and Tirole, J., 1998, 'Upgrades, Trade-ins, and Buybacks', *Rand Journal of Economics*, 29, pp. 235-258.

Hall, C. and Hall, R., 1999, 'National Policy on Microsoft: A Neutral Perspective', mimeo, <http://www.NetEcon.com>.

Katz, M. and Shapiro, C., 1986, 'Technology Adoption in the Presence of Network Externalities', *Journal of Political Economy*, 94, pp. 822-841.

Katz, M. and Shapiro, C., 1992, 'Product Introduction with Network Externalities', *Journal of Industrial Economics*, 40, pp. 55-84.

Maskin, E. and Tirole, J., 1988, 'A Theory of Dynamic Oligopoly I: Overview and Quantity Competition with Large Fixed Costs', *Econometrica*, 56, pp. 549-570.

Nahm, J., 1999, 'The Effect of Compatibility on Software Supply and the Demand for Hardware', mimeo.