Pricing Digital Marketing: Information, Risk Sharing and Performance

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Abstract: A unique value proposition of Internet-based digital marketing is the availability of precise measures of the actual performance of individual campaigns, which makes performance-based advertising pricing schedules feasible. These pricing schemes are studied in the presence of competition, performance uncertainty and asymmetric information about the quality of the client's content and the effectiveness of the publisher's technology. The paper's findings challenge the dominant practice of CPM-based pricing for digital marketing by establishing that performance-based pricing is always profit-improving for publishers, even when publishers are constrained to offer CPM-based pricing in parallel. It is shown that performance-based pricing cannot screen out clients with lower quality content/creative quality, and describe when to choose between low-end coverage and full coverage performance-based pricing. However, publishers can use performance-based pricing to credibly signal superior technological effectiveness. These results highlight the value of risk pooling for publishers of digital advertising, and the strategic role of pricing in signalling technological quality. Managerial guidelines are also provided for how to strategically respond to varying competitive intensity, client size and changes in outcome distributions.

Keywords: online advertising, information asymmetry, adverse selection, screening, signaling, electronic markets, digital goods, online marketing, banner advertising, CPM, performance-based pricing, Internet, World-Wide Web, WWW.

1. Introduction

Internet-based digital marketing has evolved from Web banner ads and text-based email marketing to include a variety of sophisticated formats, technologies and delivery mechanisms. Many digital campaigns use Shockwave and Java-enabled animations, full-screen superstitials supporting video and sound, shoshkeles which float over a web page and then collapse into a small clickable icon, and rich media email incorporating streaming audio and video. Many publishers allow marketers to place 'deep-linked' advertising text into the content of their web sites, and most search sites sell keyword-based advertising, wherein all consumer searches for a specific word or phrase trigger paid marketing messages related to that subject.

Spending on digital marketing has risen steadily over 2002, and the medium is finding its place as an integrated part of corporate marketing initiatives. Current estimates indicate that by 2006, companies in the United States will spend a total of $16 billion on online advertising, and a further $19 billion on other electronic marketing initiatives, such as email marketing and targeted promotions (Gluck, 2001).

Thus far, pricing for digital marketing has primarily been based on total quantity of impressions delivered – this impression-based method of pricing is commonly referred to as the CPM (cost per thousand impressions) model, and is a well-understood standard in the advertising industry. However, one of the unique value propositions of digital marketing is that a client can get precise measures of the actual *performance* of each campaign that they run. Examples of these performance measures include clickthrough rates on banners and icons, click rates on embedded web page hyperlinks in email campaigns, stream rates on rich media marketing messages, and response rates on keyword advertising or targeted promotions[1]. Delivery technologies like the DART system from DoubleClick support tracking detailed performance information of this kind, for both banner ads and for more sophisticated formats. Newer email marketing technologies, such as the RadicalMail suite from MindArrow Communications, can track the number of times a customer who received a rich media email streamed the embedded video. This ability to measure performance is an attractive feature to most media buyers – according to the Gartner Group, for today's cost sensitive marketers, "...accountability is more important now, and the Internet is the ultimate medium for this because of its ability to target [consumers] and measure [results]." (Gaffney, 2002).

[1] These measures may not be an exact indicator of total effectiveness – for instance, click-through rates do not measure how many people saw a banner without taking a measurable action. However, so long as they are correlated with actual effectiveness, they have value as as basis for pricing.

1.1. Performance-based pricing issues

The performance measures associated with digital marketing provide a basis for pricing models in which a client pays an amount proportionate to the actual measured performance of a marketing campaign, rather than simply paying a price based on the number of impressions delivered. The feasibility of these performance-based pricing policies clearly represents a significant opportunity for both the buyers and sellers of digital marketing. However, while some new media companies like Google have implemented sophisticated performance-based pricing systems for their advertising (Tillinghast, 2002), they are still the exception rather than the rule – in 2001, only about 20% of online advertising included some performance-based component. The limited use of performance-based pricing may be partly because designing profitable performance-based pricing structures can be challenging. Recent industry reports supports this contention, suggesting that certain kinds of pay-for-performance digital marketing are substantially mispriced. For instance, Grahn (2001) indicates that when banner ads are priced based on performance, even when using the most sophisticated tracking techniques, the delivered banner advertising inventory is undervalued by about 35%, relative to comparable CPM rates.

The complexity in developing and implementing such pricing models arises primarily from performance uncertainty, information asymmetry, risk sharing issues, and competitive pressure. These issues are elaborated on below.

Performance uncertainty and risk sharing: There is performance uncertainty associated with any marketing campaign; this stems from both the incompleteness of measurable metrics like click-through rates in actually capturing total effectiveness, as well as the inherent variability in consumer response to any specific marketing message. Any performance-based pricing structure shifts part of this performance risk away from the client and onto the publisher. Since this variability is partially determined by the composition and scope of the portfolio of campaigns that the publisher is currently exposing their audience to, publishers understand the risk better, and those with a diversified portfolio can pool performance risk. However, the extent to which these publishers should leverage their performance-based pricing models to insure client performance risk is unclear to most of them.

Variable 'creative' quality: A critical determinant of the performance of a digital marketing campaign is the quality and effectiveness of the 'creative', or the actual text, images, sound and message delivered to the audience. The quality of creative is under the control of the client (or their advertising firm), who designs it based on their knowledge of what actually comprises high-quality marketing content for their product. It is largely unknown to the publisher at the time of pricing – even if the content is available for inspection, publishers typically lack sufficient in-house expertise to assess what constitutes good creative quality (their clients are from a wide variety of product industries). As a consequence, publishers may need to design their performance-based pricing schemes to screen between high and low quality creative content, and if possible, provide incentives for the latter group to self-select CPM-based (rather than performance-based) pricing.

Effectiveness of marketing technology: There is also considerable variability in how effectively the publisher delivers a client's marketing messages. This stems from differences in technological and analytical capabilities – apart from the traditional direct marketing capabilities such as target selection (Bult and Wansbeek, 1995) and the use of purchase histories (Rossi et al., 1996), there is variability in the precision and quality of targeting technology, the ability to assess the consumer's browser and email software viewing capabilities, and the ability to time and sequence the delivery of the messages for maximal response. In addition, different publishers have access to target audience sets of different quality. A fraction of these capabilities can be assessed from repeated usage of the same publisher – however, the digital component of a marketing campaign is often bid out by large advertising firms (who own the client relationship) on a campaign-by-compaign basis, to one of a number of competing Internet marketing firms. This lack of direct and repeated interaction between clients and marketing firms increases ex-ante quality uncertainty. Consequently, publishers with superior capabilities sometimes view performance-based pricing as a way of signaling the superiority of their delivery systems, but need to ensure both that their pricing schemes make financial sense for them, and that these pricing schemes cannot be profitably imitated by lower-quality competitors.

Competitive substitutes: Online advertising and marketing is a competitive industry, and even when offering performance-based pricing, the pricing power of publishers is limited by the competitive CPM-based price, which is always an outside alternative for their potential clients. This complicates the design of an optimal pricing schedule – a problem which is further exacerbated by the fact that the marginal cost of an additional digital impression is zero. In addition, many clients continue to be uncomfortable with purely performance-based pricing schemes, because the advertising industry norm is the CPM model, and the client often has a fixed budget, or rigid media spending constraints. Consequently, in order to maintain viable business models, publishers are often forced to offer the option of a CPM-based price, which is typically the competitive rate. This means that they need to design their performance-based pricing scheme with the understanding that clients could observe, infer from, and yet bypass their performance-based pricing, while still purchasing from them by paying the competitive CPM-based price.

1.2. Research agenda and related work

These open business issues motivate this research paper, and frame its questions and agenda. Specifically, we analyze optimal performance-based pricing for digital marketing, in the presence of competition, performance uncertainty and asymmetric information about the quality of the client's content and the effectiveness of the publisher's technology. We characterize the optimal level of risk sharing in this context, and explore its profitability implications. We examine whether it is feasible and optimal for a seller to successfully screen clients with heterogeneous creative quality, and whether it is possible to use a pricing schedule to profitably signal the technological effectiveness of their online marketing systems while having to offer the CPM-based price as well. We investigate the sensitivity of these results, of seller profitability, and of value derived by clients, to changes in performance variability, client size, the intensity of competition, and the level of quality variability.

This paper represents the first analytical work on the problem of pricing digital advertising or marketing. Allied work on pricing models for advertising includes a qualitative discussion of different online advertising pricing strategies by Hoffman and Novak (2000), and some empirical studies of pricing patterns for advertising in the print media (Blankenburg, 1980, Lacy and Davis, 1991) and for television (Blank, 1968, among others). In related work characterizing advertising as an economic good, researchers have explored the nature of demand for advertising (Ehrlich and Fisher, 1982), and have shown that different types of advertising, across different media, are weakly substitutable (Silk et al., 2002). The relationship between price levels and audience demographics has been explored (Koschat and Putsis, 2000), and significant economies of scope have been associated with the services provided by advertising agencies (Silk and Berndt, 1993).

Our paper also adds to a growing literature on pricing and contracting under asymmetric information (the principal-agent literature). A central feature of many models of this kind is that contracts are characterized by risk sharing between the two parties, because this sharing of risk can mitigate some economic inefficiencies and potential market failure (Akerlof, 1970). This observation was the primary motivator of the model in this paper, since the viability of digital marketing as a business has been threatened by precisely these inefficiencies, arising from information asymmetry on both the buyer and seller sides. A prerequisite for implementing risk-sharing contracts, of course, is a robust measure of performance that can form the basis for contracting – again, a crucial differentiator of digital marketing.

There are a number of marketing papers which use the principal-agent framework. Early applications of the theory used models of 'hidden action' (moral hazard) to analyze issues in sales-force compensation (Lal and Staelin, 1985, Rao, 1990), and franchisor-franchisee relationships (Lal, 1986). A majority of the principal-agent models in the marketing literature are hidden-action models – in contrast, our model is one of 'hidden information' or adverse selection (similar to Moorthy, 1984), with a risky performance measure which can be used as the basis for contracting with risk-averse customers. Papers from the literature that have applied hidden-information models most related to ours include Lutz and Padmanabhan (1995), who analyze the optimal insurance (warranty) levels for products purchased by risk-averse customers, Png (1989), who characterizes the role of reservations in insuring risk-averse customers who face uncertain product availability and are uncertain about their valuation for the product, and Moorthy and Srinivasan's (1995) model of using money-back guarantees to signal product quality. A related paper is Desai and Srinivasan (1995), who integrate a hidden information model with one of moral hazard to characterize how a franchisor can signal quality (demand potential) to an uninformed franchisee.

Since our model is designed specifically for our context – pricing digital marketing – it is analytically distinct from the ones already in the literature, most saliently in its representation of competition, and in the constraints placed on offering CPM-based pricing in addition to a performance-based contract. There has been some qualitative discussion of the potential of applying a principal-agent model to study performance-based contracting for advertising services (Ellis and Johnson, 1993, Spake et al., 1999), and our paper represents an actual modeling effort of this kind.

Our model is somewhat different from other work on advertising signals of quality (work similar to Milgrom and Roberts, 1986, for instance), in that pricing in our model is for the advertising itself, rather than for the good being advertised, and in that the realized performance of the advertising can be incorporated into a contract – it is the absence of the latter feature which necessitates either a multi-period model, or 'money burning' signals of quality in the usual advertising signaling models.

The rest of the paper is organized as follows. Section 2 describes the basic model, derives the optimal pricing schedule under symmetric information, and provides a number of comparative statics results, many of which apply to subsequent propositions in Section 3. The next section extends the basic model by incorporating two types of asymmetric information, analyzing performance based pricing in the presence of potential adverse selection due to client quality uncertainty, and the feasibility of heterogeneous publishers signaling their technological effectiveness. Section 4 discusses the managerial implications of the paper's results, places them in the existing related literature, and outlines current research.

2. Model

The first part of this section provides an overview of the basic model used in the paper. The second part introduces the graphical representation of the problem space that we use to illustrate our results. The final part derives the optimal performance-based pricing under symmetric information, analyzes the sensitivity of pricing and profits to the model's parameters, and discusses the implications the model's assumptions.

2.1. Model Overview

The focus of the model is on the pricing decisions made by a single firm, also referred to as the *publisher*, who provide a homogeneous digital marketing service (referred to as the *service*, or simply as *marketing*) to its clients. Each client may purchase upto one unit of this marketing service. Examples of a unit of marketing are a fixed number of banner ad exposures, or a fixed number of targeted email messages. For clarity, we refer to the buyer of the marketing service as the *client*, and the people who are exposed to the marketing as the *consumers*. Consequently, our model examines contracts between *publishers* and their *clients*.

2.1.1. Outcomes and performance

The measurable performance of each unit of digital marketing is represented by a random variable $\mathbf{z}$ which is referred to as the *outcome*, and which has support Z. Examples of $\mathbf{z}$ include the number of clickthroughs on a banner ad, the number of clicks on an embedded hyperlink in an email message, the response rate on a targeted promotion, or the total number of times an embedded video is streamed to consumers receiving a rich-media email message. A *pricing scheme* (also called a *contract*) is a function $t(.)$ that defines a specific payment $t(z)$ from the client to the publisher for each point $z \in Z$. In general, a pricing scheme is therefore performance-based, unless $t(.)$ is constant.

2.1.2. Clients

Clients have identical Bernoulli utility functions[2] $u(.)$ defined over certain monetary outcomes. $u(.)$ is continuous, thrice-differentiable, strictly increasing and strictly concave. Consequently, clients are

[2] Our definitions of the terms Bernoulli utility function and von-Neumann-Morgernstern expected utility function are as in Mas-Colell, Whinston and Green, 1995, Chapter 6.

risk-averse. In addition, $u'(.)$ is strictly convex[3]. Each client also has a deterministic profit level w independent of the outcome of the marketing service. w can be interpreted as representing the size of a client's overall marketing budget, or of their overall business. A client's expected value of a unit of marketing is therefore given by their von-Neumann-Morgernstern expected utility $E[u(w + \mathbf{z} - t(\mathbf{z}))]$, where the expectation is taken over Z. For simplicity, we have rescaled monetary units so that the outcomes $z \in Z$ have unit variable monetary value. We have also assume that the client faces no threat of bankruptcy as a consequence of their digital marketing effort – that is, $w + z - t(z) > 0$ for all $z \in Z$.

If the publisher offers multiple pricing schemes, the client considers the pricing scheme $t(.)$ which maximizes $E[u(w + \mathbf{z} - t(\mathbf{z}))]$ – this is called the *most favorable* pricing scheme.

2.1.3. Exogenous competition

Competition is represented by the presence of an alternate marketing service with an exogenously specified non-performance-based pricing scheme $t(z) = p$. This is analogous to an 'outside good' (as in Salop, 1979, for instance). The presence of this alternative service is assumed to affect client behavior in the following way:

1. If the publisher's most favorable pricing scheme $t(.)$ is such that

$$E[u(w + \mathbf{z} - t(\mathbf{z}))] < E[u(w + \mathbf{z} - p)], \tag{2.1}$$

   then each client purchases one unit of the alternative marketing service.

2. If the publisher's most favorable pricing scheme $t(.)$ is such that

$$E[u(w + \mathbf{z} - t(\mathbf{z}))] \geq E[u(w + \mathbf{z} - p)], \tag{2.2}$$

   then each client purchases one unit of marketing from the publisher, and the publisher's expected revenue per client is $E[t(\mathbf{z})]$

[3]That is, $u'''(x) > 0$. This places some structure on the behavior of $u'(x)$ – since it is strictly positive and strictly decreasing, one would expect it to exhibit 'convex behavior' in a global sense: if, on the contrary, $u'(x)$ decreased at a decreasing rate for a significant range of x – that is, if it's slope $u''(x)$ became increasingly negative as x increased – then $u'(x)$ would eventually become negative. The assumption ensures that it exhibits this appropriate behavior locally as well. It is satisfied by commonly used concave functions that are strictly increasing. For instance, all concave functions of the form $a + bx^c$ as well as all functions of the form $a + b\log(x^c)$ satisfy this assumption.

3. If the publisher's most favorable pricing scheme is $t(z) = p$, then a client purchases one unit of marketing from the publisher[4], and the publisher's revenue per client is p.

Assumption (3) is implied by assumption (2), but is reiterated for clarity. This set of assumptions simply specify that the client must expect at least as much utility from the publisher as they get from the alternative service, and that the publisher always has the option of selling their marketing service at the competitive price p. One could therefore think of the publisher as the weakly preferred provider in a competitive market. A decrease in p represents an increase in competition, and consequently, more 'external pricing pressure' on the publisher.

2.1.4. Pricing schemes and publisher profits

The publisher is restricted to offering linear pricing schemes of the form $t(z) = \alpha + \beta z$, where $\alpha \in [0, p]$ and $\beta \in [0, 1]$. The publisher may offer just the competitive pricing scheme $t(z) = p$, which is not based on performance, or may offer the competitive pricing scheme in addition to a single performance-based pricing scheme. The restriction on the range of α and β is to ensure that publishers offer pricing schemes that are realistic. For example, $\beta > 1$ would correspond to a pricing scheme in which the publisher offers insurance over and above the value of the marketing service, which is not consistent with offers that are feasible in practice. Similarly, $\alpha < 0$ would imply that the publisher transfers money to the client in exchange for their business. The implications of these restrictions, as well as those of the restriction of linearity, are discussed further in section 2.3.

For expositional simplicity, we sometimes refer to a pricing scheme as the pair (α, β). In line with our discussion in section 1.1, the publisher is assumed to be risk neutral, and the marginal cost of offering the service is zero. The publisher's payoff per unit of marketing sold is therefore $E[\alpha + \beta \mathbf{z}]$. The publisher's objective is to maximize these expected profits.

2.2. Graphical representation of outcomes

The results of this paper are all presented in the context of a two-outcome[5] model $Z = \{z_L, z_H\}$, where $z_L < z_H$. In this context, the distribution of $\mathbf{z}$ is represented by the probability π of the high outcome z_H. When there are multiple types of clients or publishers, this is indicated by a superscript

[4] One might argue that choice should be probabilistic in the case of the client being indifferent (a la Bertrand). However, this add considerable analytical complexity by introducing discontinuities in the publisher's payoffs, and requiring optimal pricing schemes to be ε away from the indifference point, with very little change in the business implications of the results.

[5] Generalizations to the case of a continuum of outcomes are dicsussed for some results.

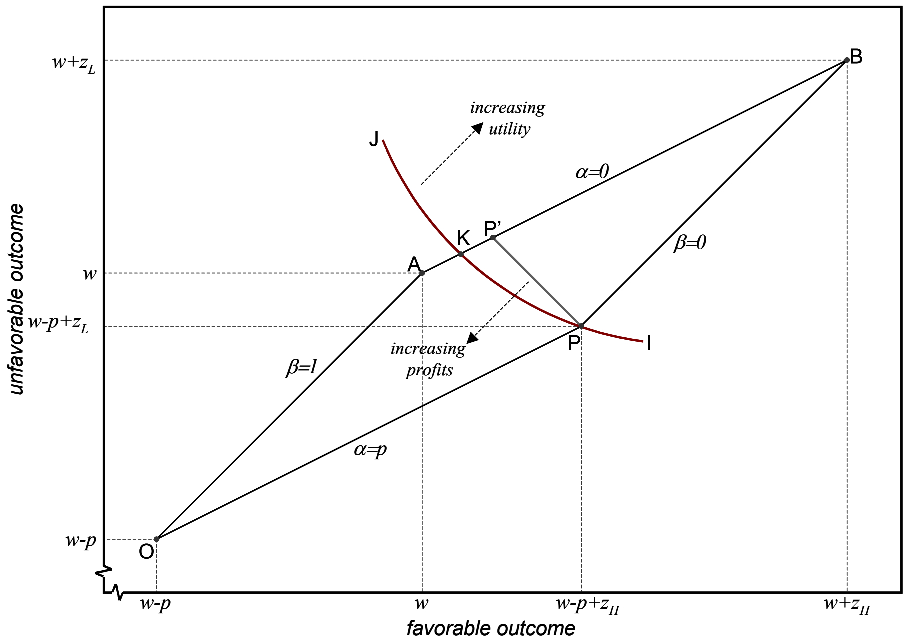

Figure 2.1: The set of feasible pricing schemes. $OABP$ represents the set of possible contracts, IJ is the client's indifference curve, and PP' is the publisher's isoprofit line.

on π.

This simplification allows us to easily represent the problem space on a two-dimensional graph, as illustrated in Figure 2.1. This kind of representation is commonly used in the literature on contracting under asymmetric information and uncertainty (see, for instance, Stiglitz, 1977, for a detailed step-wise exposition, and Riley, 2001, for examples of their usage, and references to other work). The axes measure the (certain) total client payoffs in the case of each of the two outcomes, net of payments to the publisher. Consequently, a pricing scheme that charges p independent of outcome is represented by the point P in Figure 1. At this point P, if the outcome is favorable (z_H), then the client gets a payoff of z_H (from the marketing), plus w (certain independent profits), and minus p (price paid to publisher), resulting in a net monetary payoff of $w - p + z_H$. Similarly, if the outcome is unfavorable, the client gets a net monetary payoff of $w - p + z_L$.

The client payoffs under any feasible pricing scheme (α, β) are represented in Figure 2.1 by points in the parallelogram $OABP$. The point A corresponds to a pricing scheme ($\alpha = 0, \beta = 1$), which leaves the client with simply their certain profits w. Similarly, the point B corresponds to ($\alpha = 0, \beta = 0$) – this is when the marketing has a price of zero – and the point O corresponds to ($\alpha = p, \beta = 1$). The line AB represents the set of contracts for which $\alpha = 0$, and as one moves from B towards A, β increases from 0 to 1. Similarly, the line PB represents the set of contracts for which $\beta = 0$, and

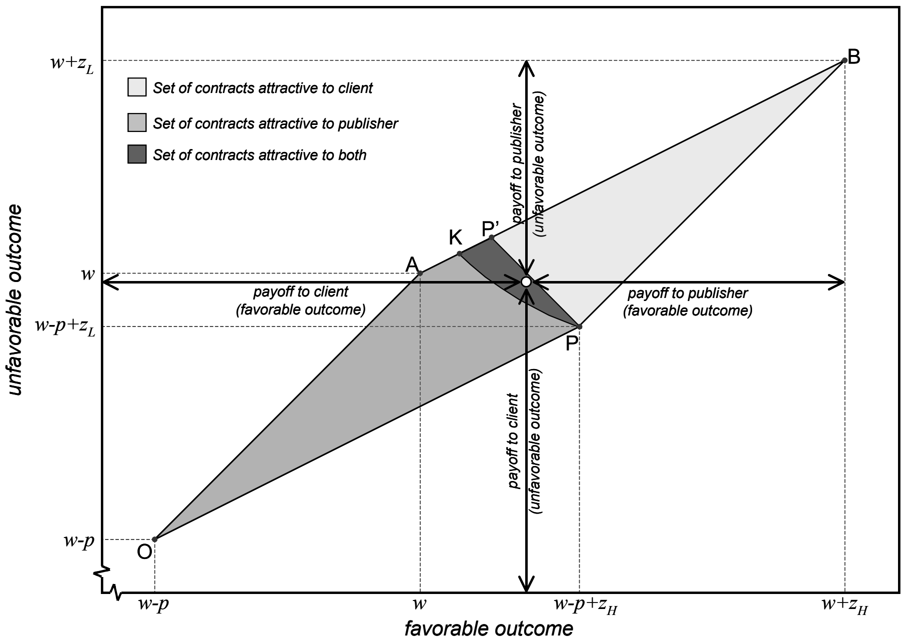

Figure 2.2: $KP'P$ represents the pricing schemes that are simultaneously profit-improving for the publisher, and acceptable to the client – the intersection of the sets $OAP'P$ and KBP. The labeled arrows represent the magnitude of the payoffs to each party under each outcome, for a candidate contract in $KP'P$.

as one moves from B to P, α increases from 0 to p. Any point in the interior of $OABP$ represents a pricing scheme for which $0 < \alpha < p$, and $0 < \beta < 1$.

The client's indifference curves in this space illustrate the relative value clients place on the different pricing schemes. Clearly, the slope of these indifference curves is negative; moreover, so long as $0 < \pi < 1$, the fact that $u(.)$ is strictly concave implies that the slope of the client's indifference curves is strictly increasing (and therefore strictly less negative) as one moves to the right. This is explained analytically in the proof of Lemma 1. The indifference curve IJ passing through P is illustrated in Figure 2.1. The direction of increasing utility is upwards and to the right.

The line PP' is an isoprofit line for the publisher. Given the representation of contracts, the publishers's payoff from any point (x, y) on the plane is $(w + z_H - x)$ in the case of a favorable outcome, and is $(w + z_L - y)$ in the case of an unfavorable outcome. The isoprofit line through P has a slope of $-\pi/(1 - \pi)$, and this slope is the same at any point on the plane. The direction of increasing profits is to the left and downwards.

Consequently, while the entire region $OABP$ is feasible, the presence of the competitive price p restricts the set of relevant contracts significantly. The client will not be interested in any contract that lies below IJ – the set of contracts that the client will consider relative to P is in the area KBP

in Figure 2.2. Similarly, the publisher will not want to offer any contract that lies above PP', since this strictly reduces their expected payoff, relative to P. The set of profit-improving contracts for the publisher is therefore in the area $OAP'P$. The set of contracts of interest to both contracting parties is consequently the intersection of KBP and $OAP'P$, which is the area $KP'P$, as illustrated in Figure 2.2. The payoffs to the client and to the publisher under each outcome are also illustrated for a candidate contract in this region.

2.3. Optimal pricing under symmetric information

This subsection establishes the existence of a performance-based pricing scheme that improves the publisher's expected profits, proves that the unique optimal pricing scheme is purely performance-based, and derives some comparative statics results that apply to the base model as well as some of the extensions in Section 3.

2.3.1. Existence

As indicated in Figure 2.2, the region $KP'P$ contains pricing schemes that are preferred by the client over P, and that increase the publisher's expected payoff. Consequently, establishing existence is merely a matter of showing that the region $KP'P$ has a strictly positive area, which is done in Lemma 1. All proofs are in Appendix A.

Lemma 1. *If $0 < \pi < 1$, there exists a non-empty set of pricing schemes $G \subset [0, p] \times [0, 1]$ such that for any $(\alpha, \beta) \in G$:*

$$\alpha + \beta[\pi z_H + (1 - \pi)z_L] > p, \tag{2.3}$$

and

$$E[u(w + (1 - \beta)\mathbf{z} - \alpha)] \geq E[u(w + \mathbf{z} - p)]. \tag{2.4}$$

The proof of Lemma 1 also formalizes some of the discussion in Section 2.2 – specifically, the shape and slope of the client's indifference curve, the profit-improving direction for the publisher, and the fact that the slope of the isoprofit line is $-\dfrac{\pi}{1-\pi}$.

2.3.2. Unique optimal pricing scheme

The publisher will choose the pricing scheme in G above that maximizes her expected payoff.

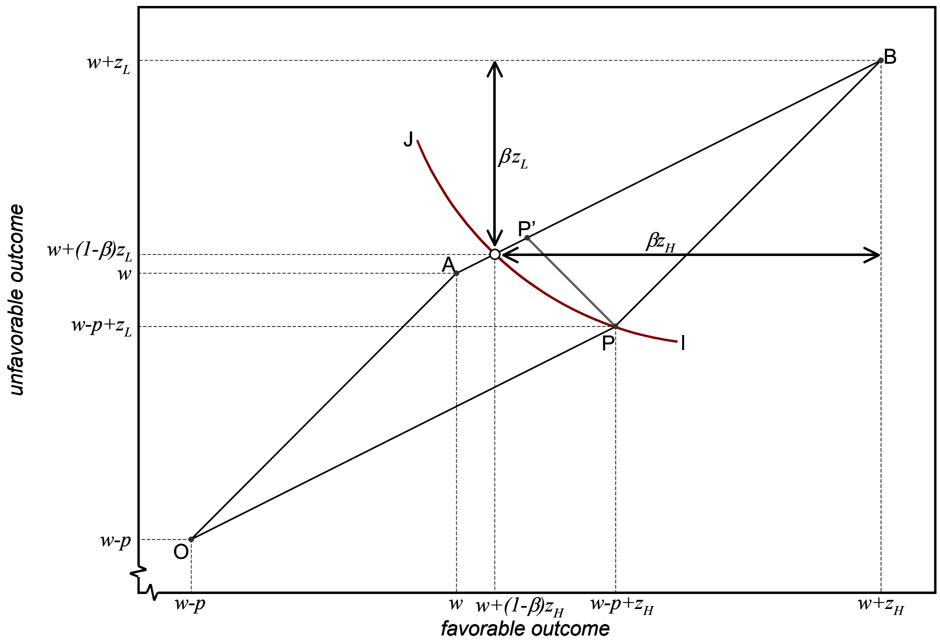

Figure 2.3: The optimal performance-based pricing scheme, at the intersection of AB and IJ.

Proposition 1. *The optimal performance-based pricing scheme* $(\alpha^*, \beta^*) \in G$ *satisfies:*

$$\alpha^* = 0 \tag{2.5}$$

and

$$E[u(w + (1 - \beta^*)\mathbf{z})] = E[u(w + \mathbf{z} - p)]. \tag{2.6}$$

The latter equation can be rewritten as:

$$\frac{u(w + (1 - \beta^*)z_L) - u(w + z_L - p)}{u(w + z_H - p) - u(w + (1 - \beta^*)z_H)} = \frac{\pi}{1 - \pi}. \tag{2.7}$$

The optimal pricing scheme is illustrated in Figure 2.3. Intuitively, the isoprofit line QQ' that passes through the point K corresponding to the optimal pricing scheme is as 'far away' from PP' as possible, while still intersecting the set of relevant pricing schemes[6]. Also, since $u(.)$ is strictly increasing, the numerator of the LHS of (2.7) is strictly decreasing in β, while the denominator is strictly increasing, which implies that the expression is strictly decreasing. Since the RHS is constant

[6] The proof of Proposition 1 is constructed in a manner that does not use the fact that the support Z of $\mathbf{z}$ has two elements (or that Z is finite, for that matter), and therefore generalizes to any distribution (with finite or continuous support) of a one-dimensional uncertain performance measure $\mathbf{z}$, though the expression (2.7) will be different for different distributions.

for a fixed π, and given Lemma 1, this is a simple way of verifying that β^* is indeed unique.

The fact that the optimal pricing scheme involves no fixed component may be puzzling in light of the fact that many performance-based pricing schemes in practice have both a CPM and a performance-based component to them. However, $\alpha^* = 0$ does not necessarily indicate that the pricing scheme will be presented as a purely performance-based one. In our two-outcome model, for example, it is certain that the client will pay the publisher at least $\beta^* z_L$. Hence, the pricing scheme can be thought of and presented as involving a fixed component of $\beta^* z_L$, and a performance-based fee of $\beta^*(z_H - z_L)$, payable in the event of a successful outcome. Clearly, this generalizes to a multi-outcome model so long as the least favorable outcome has a positive payoff. Often, pricing schemes are presented by sellers in a manner that can be benchmarked with comparables most easily – and the typical client may be used to a structure that includes a CPM-based component.

2.3.3. Comparative statics and discussion

Based on Proposition 1, we explore the sensitivity of the optimal performance-based pricing scheme and of publisher profits to changes in some of the model's parameters. These comparative statics results are summarized in Propositions 2 through 4. The results use the fact that the publisher's expected profit per unit of marketing sold under the optimal performance-based pricing scheme is:

$$E[\beta^* \mathbf{z}] = \beta^*(\pi z_H + (1 - \pi) z_L). \tag{2.8}$$

The parameters p and w are analyzed first. These are the parameters which when varied, affect the publisher's profits only through their effect on β^*.

Proposition 2. *(a) The optimal performance-based price $\beta^*$is strictly increasing in the competitive CPM-based price p:*

$$\frac{\partial \beta^*}{\partial p} = \frac{E[u'(w + \mathbf{z} - p)]}{E[\mathbf{z}u'(w + \mathbf{z}(1 - \beta))]} > 0. \tag{2.9}$$

(b) The profits from a performance-based pricing scheme are strictly increasing in p, and are more sensitive to changes in p than the corresponding profits from a CPM-based pricing scheme:

$$\frac{d}{dp} E[\beta^* \mathbf{z}] = \frac{E[u'(w + \mathbf{z} - p)].E[\mathbf{z}]}{E[\mathbf{z}u'(w + \mathbf{z}(1 - \beta))]} > 1. \tag{2.10}$$

Part (a) of Proposition 2 is not surprising. An increase in p corresponds to a reduction in the intensity of competition, and a decrease in client utility from the competitive alternative. Consequently,

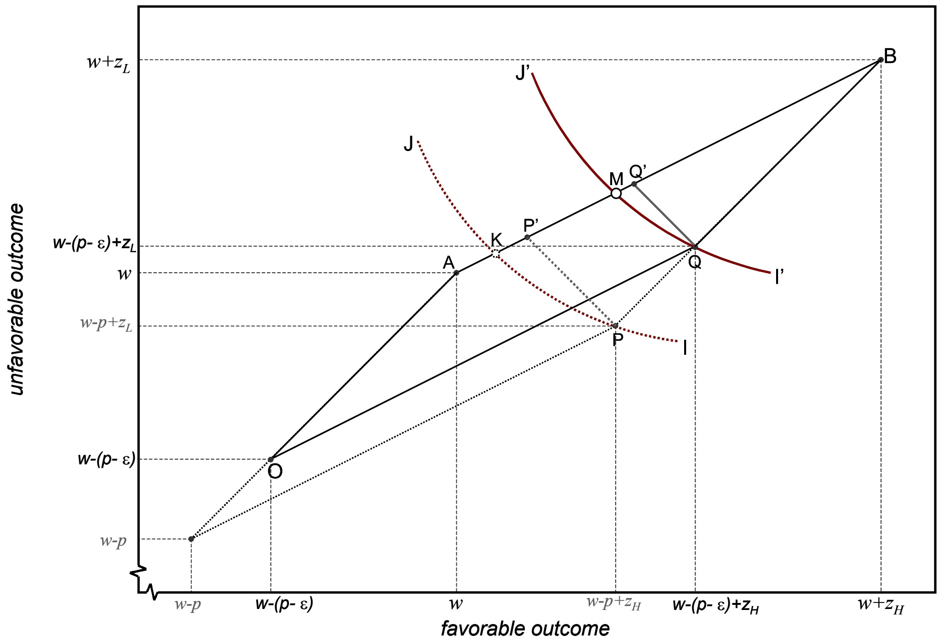

Figure 2.4: The impact of an small decrease ε in the competitive price p. The change in p shifts the optimal pricing scheme from K to M, resulting in an expected profit increase that is more than ε.

one would expect both the performance-based price and the resulting profits to increase. A decrease in p on the other hand represents an increase in competition, and in client utility, and consequently, a lower β^*.

However, part (b) of the proposition implies that an increase in p benefits publishers who offer performance-based pricing *more* than publishers who are actually selling at the competitive price p (since the expected profits per unit of marketing from the price p are simply p, and a unit increase in p causes a unit increase in these profits). Figure 2.4 illustrates the change in the optimal contract for a decrease of ε in p, and provides some intuition. When the price p falls, the client's base indifference point O shifts upwards and to the right by ε, and the relevant indifference curve is now $I'J'$. The publisher's expected profits from offering the competitive price shift accordingly, and the net reduction in expected profits is also ε. However, the shift results in a decrease in the slope of the client's indifference curve at Q (relative to P), and at every point in the feasible region (that is, the slope of the curve $I'J'$ is more negative at every point along QM, relative to the slope of the curve IJ at the corresponding points on PK). This means that the distance between K and M (which is proportionate to the decrease in expected profits from the performance-based pricing scheme) is higher than the distance between P' and Q' (which is proportionate to ε, the decrease in expected profits from offering the price p). The opposite argument holds for an increase in price – the shift from M to K is more than the shift from Q' to P'.

A qualitatively similar effect occurs when w is changed:

Proposition 3. *The optimal performance-based price β^* and the corresponding publisher profits $E[\beta^* \mathbf{z}]$ are both strictly decreasing in w.*

Proposition 3 indicates that the performance-based price and corresponding profitability will be lower per unit of marketing from larger clients – that is, clients whose spending on digital marketing is a smaller fraction of their overall profits or total marketing budget. Note that this proposition compares pricing across the *same quantity* of marketing (and all clients are assumed to have available the same CPM price p available to them)– the result reflects the fact that smaller clients face higher total variability per unit of uncertain marketing performance, and consequently, can be charged higher prices under performance-based contracts that reduce this variability.

Next, we explore the impact of changing the distribution of the outcome $\mathbf{z}$. This kind of change affects the publishers profits both directly through its effect on $E[\mathbf{z}]$, and indirectly through its effect on β^*. We consider two changes explicitly – a constant shift in the support of $\mathbf{z}$, which increases z_L and z_H by equal amounts, and consequently increases the mean $E[\mathbf{z}]$ by the same amount, but does not change the variance of $\mathbf{z}$ – and an increase in just z_H, which increases both the mean and the variance of $\mathbf{z}$. The changes have been chosen to contrast their effects on firm profits. We also briefly discuss the implications of changing π (which affects mean, variance and skewness).

Proposition 4. *(a) An increase in the average level of marketing performance $E[\mathbf{z}]$ via a simultaneous increase in both z_L and z_H reduces the optimal performance-based price β^*, and also reduces the corresponding profits $E[\beta^* \mathbf{z}]$.*

(b)An increase in the mean and variance of marketing performance via an increase in just z_H increases the profits $E[\beta^* \mathbf{z}]$ from the performance-based pricing scheme.

An increase in the mean of z with no change in other parameters will naturally cause the per-outcome rate β^* to fall. What is surprising is that the reduction in β^* is more significant than the increase in $E[\mathbf{z}]$, resulting in a drop in overall profits from the performance-based price. This is an example of where the modeling of a competitive alternative service plays a crucial role. Intuitively, the performance-based price has to be set relative to the utility the client gets from this alternative service, which is measured at the CPM-based price p. This 'reservation utility' increases when $E[\mathbf{z}]$ increases, and increases more rapidly than the client's utility from the existing performance-based pricing scheme (since $\beta^* > 0$). In compensating for this, and ensuring that the client still chooses the performance-based pricing scheme over the alternative service, the publisher's profits fall, and the publisher gets none of the benefits from superior performance.

Part (b) of the proposition highlights the role that performance risk plays in the performance-based pricing scheme. The increase in variance that accompanies the increase in mean compensates for the increase in 'reservation utility' described above – resulting in overall higher profits for the publisher.

Finally, an increase in the probability of a favorable outcome π reduces the performance-based price β^*:

Corollary 1. *The optimal performance-based price β^* is strictly decreasing in π.*

2.3.4. Further observations

The impact of changes in π on publisher profits is not unambiguously clear. An increase in π increases the mean of z, and increases the expected utility of clients This makes the client better off, and always lowers the optimal value of β^*– this is intuitively clear if one notes that as π increases, the slope of the client's utility function becomes more negative and rotates to the right about P. The effect on the profits of the publisher appears to be positive for low values of π, and negative for high values of π.

As promised at the beginning of section 2, we now discuss the implications of the restrictions on the values of α and β, and the assumption of linearity. In the context of our two-outcome exposition, linearity places no real restriction on contracts – since we consider all possible pairs of client-publisher payoff in $OABP$ as candidate solutions, any non-linear contract (that is, one that charges the client different fractions for favorable and unfavorable outcomes) can simply be restated as a linear contract by appropriately adjusting the fixed payment α.

However, the restrictions on the parameter values of α and β are binding. In their absence, the optimal pricing scheme would be $\beta = 1$ and $\alpha < 0$ – where the publisher takes on all performance risk, and pays the client their certainty equivalent of the random outcome $\mathbf{z}$ less the competitive price p – this is illustrated in Figure 2.5. While this is a familiar solution in the standard insurance context (a premium in exchange for bearing risk), this is not a realistic outcome in the context of digital marketing, and we consequently feel that our restrictions are justified in the interest of making our results more relevant to our specific business problem.

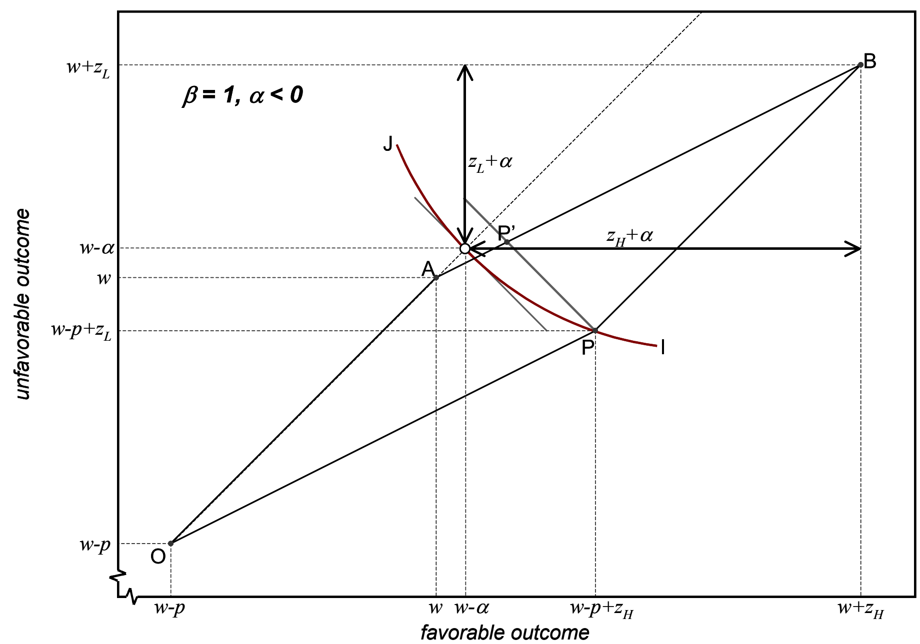

Figure 2.5: The unconstrained optimal 'fully insured' contract, which lies outside the feasible region $OABP$. The extension of the line OA is the 45-degree line through the origin.

3. Client quality and technological uncertainty

The base model presented in Section 2 is enhanced in this section to allow multiple types of clients, with unknown quality of marketing content, and to also allow multiple types of publishers, with differing technological effectiveness. Each subsection in this section defines its own independent modifications to the base model.

3.1. Screening clients with uncertain creative quality

The first part of this section describes the modifications of and additions to the parameters of the base model, needed in order to incorporate uncertain client quality. The second part establishes that the publisher cannot successfully design a performance-base pricing scheme that screens out the lower quality clients, and induces just the higher quality clients to accept performance-based pricing. Following this, the third section examines the alternatives that the publisher has – a performance-based pricing scheme that is adopted by all clients, or one that is adopted by just the lower-quality clients – by characterizing the relevant pricing schemes, and analyzing their profitability and sensitivity to the model's parameters.

3.1.1. Modifications and additions to base model

The structure of the model described in Section 2.1 is preserved for the most part. However, clients are now of two types $\theta \in \{l, h\}$. Type-h clients are sometimes referred to as *high-quality* clients, and type-l clients as *low-quality* clients. Both clients have the same Bernoulli utility function $u(.)$ with the properties described in section 2.1. The client types differ in their relative probabilities of favorable and unfavorable outcomes. Specifically, the performance of a unit of marketing for a client of type θ has an outcome represented by the random variable $\mathbf{z}^\theta$ with common support $Z = \{z_L, z_H\}$, and the probability of outcome z_H for a client of type θ is π^θ, where $0 < \pi^l < \pi^h < 1$. We assume that the marketing service is valuable to both client types at the competitive price, which implies that $E[\mathbf{z}^\theta] > p$. The fraction of type h clients in the publisher's market is λ, and the fraction of type l clients is $(1 - \lambda)$.

The values of all these parameters is common knowledge. However, when offering a client a performance-based pricing scheme, the publisher does not know what type the client is. Therefore any pricing scheme offered by the publisher may be chosen by either type of client, and both types of clients also have access to the same competitive price p. Clients make their choices as per equations (2.1) and (2.2), where expectations are taken by the client with full knowledge of their own type.

The publisher may offer the competitive pricing scheme $t(z) = p$, and may also offer one performance-based pricing scheme $t(z) = \alpha + \beta z$, with the same restrictions on the values of α and β. We discuss the implications of offering multiple pricing schemes in Section 4. The publisher maximizes expected profits, taking into account information asymmetry and the behavior of each client type – the expressions for expected profits are presented in equations (3.7) and (3.8).

3.1.2. Separating client types

Figure 3.1 illustrates the indifference curves and the corresponding isoprofit lines for the two types of clients. IJ is the indifference curve of the type-l client, and KL is the indifference curve of the type-h client.

The following proposition establishes that the publisher cannot design a performance-based pricing schedule that is selected only by the type-h clients:

Proposition 5. *If a feasible performance-based pricing scheme (α, β) that is profit improving for any type of client is selected by the high-quality clients, then it will also be selected by the low-quality*

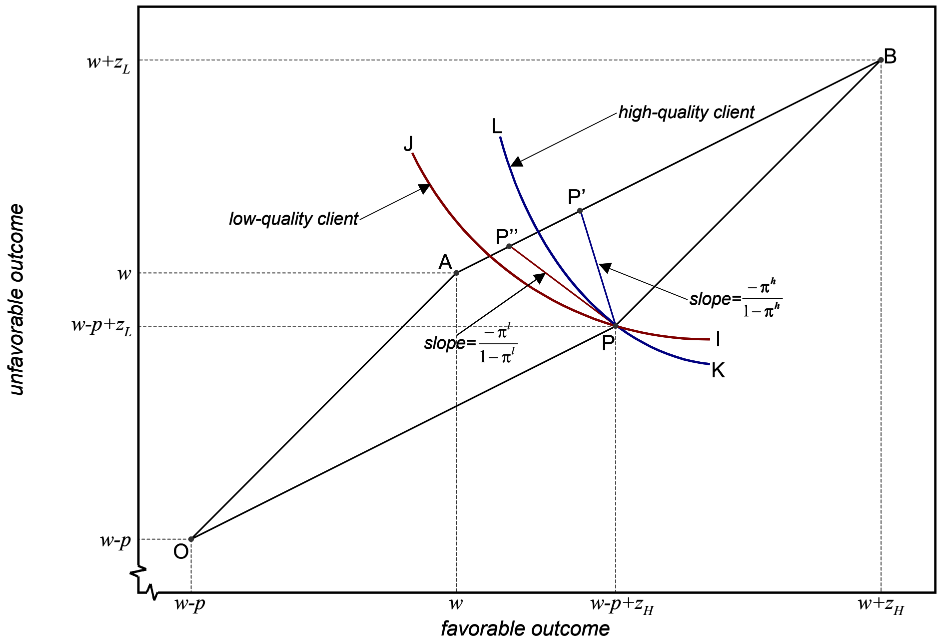

Figure 3.1: The indifference curves KL and IJ of the high and low-type clients respectively. The corresponding isoprofit lines for the publisher are PP' and PP''.

clients. That is, if $(\alpha, \beta) \in [0, p] \times (0, 1]$ *is such that:*

$$\pi(\alpha + \beta z_H) + (1 - \pi)(\alpha + \beta z_L) \geq p \text{ for any } 0 < \pi < 1, \tag{3.1}$$

and

$$E[u(w + (1 - \beta)\mathbf{z}^h - \alpha)] \geq E[u(w + \mathbf{z}^h - p)], \tag{3.2}$$

then

$$E[u(w + (1 - \beta)\mathbf{z}^l - \alpha)] > E[u(w + \mathbf{z}^l - p)]. \tag{3.3}$$

Under one more concavity assumption on $u(.)$, the above result generalizes to outcome distributions with a continuous support (where a higher-quality client is one whose outcome distribution first-order stochastically dominates that of the lower-quality client) – a proof of this is available on request.

The shaded region in Figure 3.2 illustrates the set of contracts which will be preferred over P by the high-quality clients, but not by the low-quality clients. Clearly, this entire set is outside the feasible region. However, even if one relaxed the constraints on α and β, the result would not change. This is because all the contracts in the shaded region also strictly reduce the publisher's profits relative to those obtained by offering p.

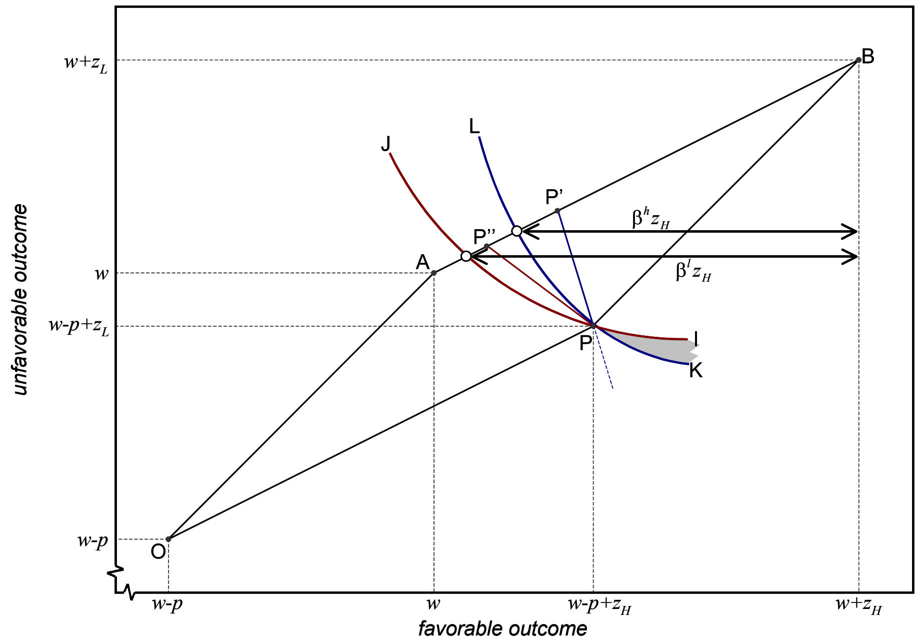

Figure 3.2: The first-best contracts for each client type, at intersection of AB and the indifference curve IJ (type-l) and KL (type-h). The shaded region IPK represents contracts that could screen type-l clients – however, none of these is either feasible or profitable.

3.1.3. Optimal performance-based pricing schemes

Before analyzing the publisher's options further, we define the first-best contracts – these are the pricing schemes that the publisher would offer each client type if the entire population consisted of just that client type. By Proposition 1, these are purely performance-based prices – denote them β^h and β^l respectively:

$$E[u(w + (1 - \beta^h)\mathbf{z}^h)] = E[u(w + \mathbf{z}^h - p)], \tag{3.4}$$

and

$$E[u(w + (1 - \beta^l)\mathbf{z}^l)] = E[u(w + \mathbf{z}^l - p)]. \tag{3.5}$$

These pricing schemes are indicated in Figure 3.2. Clearly, $\beta^l > \beta^h$.

When the publisher is constrained to offering just one performance-based pricing scheme, Proposition 5 implies that the publisher has three options – selling at the competitive pricing scheme p to both client types, offering a performance-based pricing scheme that will be selected by only the low-quality types, or offering a performance-based pricing scheme that is selected by both client types. The next result shows that the publisher always chooses one of the first-best contracts β^h or β^l.

Proposition 6. *Under uncertain client quality, the publisher always offers a performance-based pric-*

ing scheme, and chooses either β^h or β^l. Moreover, under any set of parameters, there is always a critical fraction λ^* of high-quality customers:

$$\lambda^* = \frac{(\beta^l - \beta^h)E[\mathbf{z}^l]}{(\beta^l - \beta^h)E[\mathbf{z}^l] + (\beta^h E[\mathbf{z}^h] - p)} \tag{3.6}$$

such that if $\lambda < \lambda^*$, the publisher offers β^l, and the profit per unit of marketing is

$$\Pi_{C1} = \lambda p + (1-\lambda)\beta^l E[\mathbf{z}^l], \tag{3.7}$$

and if $\lambda > \lambda^*$, the publisher offers β^h and the profit per unit of marketing is

$$\Pi_{C2} = \beta^h(\lambda E[\mathbf{z}^h] + (1-\lambda)E[\mathbf{z}^l]) \tag{3.8}$$

The subscript C on Π is to indicate that this is a payoff associated with *client* quality uncertainty. Note that since $\beta^h E[\mathbf{z}^h] > p$, the RHS of (3.6) is strictly less than 1, and none of the terms in the expression depend on λ. This means that as the fraction of high-quality clients increases, there is always a point at which the publisher switches to offering β^h. If $\beta^h E[\mathbf{z}^l] \geq p$, then the performance-based price β^h is profit-improving even when adopted by the low-quality clients. However, when $\beta^h E[\mathbf{z}^l] < p$, and the publisher offers β^h, then the publisher loses money on the type-l clients, relative to selling at the competitive price p – this is a cost borne by the publisher as a consequence of asymmetric information.

Also note that the high-quality customers never get higher utility than $E[u(w+\mathbf{z}^h - p)]$. However, when $\lambda > \lambda^*$, the low-quality customers get a strictly positive increase in utility. This is likely to exacerbate the problem of unknown client quality. The low-quality clients can benefit from the publisher believing that there is a higher fraction of high-quality clients in the market, and consequently have an incentive to pro-actively shield the true quality of their creative content.

If $\lambda \leq \lambda^*$, and the publisher's payoff is Π_{C1}, then all the comparative statics results from section 2.3 continue to hold. For instance:

$$\frac{d\Pi_{C1}}{dp} = \lambda + (1-\lambda)\frac{\partial \beta^l}{\partial p}E[\mathbf{z}^l] > 1, \tag{3.9}$$

since we know from Proposition 2 that $\frac{\partial \beta^l}{\partial p}E[\mathbf{z}^l] > 1$. Similarly

$$\frac{d\Pi_{C1}}{dw} = (1-\lambda)\frac{\partial \beta^l}{\partial w}E[\mathbf{z}^l] < 0, \tag{3.10}$$

since we know from Proposition 3 that $\frac{\partial \beta^l}{\partial w} E[\mathbf{z}^l] < 0$. The results are analogous for changes in z_L and z_H. While directionally similar, the magnitude of the effect of changes in these parameters on publisher payoffs will be less pronounced, since only a fraction of their clients adopt the performance-based pricing scheme. If $\lambda \geq \lambda^*$, all the comparative statics results about the value of the performance-based price continue to hold, but not those related to profits.

Further insights based on these results are presented in Section 4.

3.2. Signaling technological effectiveness

We now turn to the problem of modeling asymmetric information about the effectiveness of the system that the publisher uses to deliver their digital marketing service. The first part of this section describes the modifications and additions to the base model, towards incorporating uncertain technological effectiveness. The second part establishes that a more effective publisher can always use their performance-based pricing scheme to signal the fact that their technological effectiveness is higher, examines the two different constraints that the publisher faces when choosing their separating performance-based pricing scheme, and describes the sensitivity of pricing and profits to the model's parameters.

3.2.1. Modifications and additions to base model

As in section 3.1, the structure of the model described in Section 2.1 is largely preserved. In this section, however, *publishers* are of two types $\theta \in \{l, h\}$. Type-h publishers are sometimes referred to as *superior* publishers, and type-l publishers as *inferior* publishers. The publisher types differ in the relative probability of favorable and unfavorable outcomes from the marketing that is delivered by their systems. Using a superior publisher's online marketing system results in a higher probability of a favorable outcome, for any client. Specifically, the performance of a unit of marketing sold by a publisher of type θ has an outcome represented by the random variable $\mathbf{z}^\theta$ with support $Z = \{z_L, z_H\}$. The probability of outcome z_H from using the marketing system of a publisher of type θ is π^θ, where $0 < \pi^l < \pi^h < 1$. We continue to assume that the marketing service of both publishers is valuable at the competitive price, which means that $E[\mathbf{z}^\theta] > p$.

The publisher may offer one performance-based pricing scheme $t(z) = \alpha + \beta z$, with the same restrictions on the values of α and β. In addition, we require that the publisher <u>must offer</u> their service at the competitive price p as well. The assumption is motivated by the reality that most publishers need to offer a CPM-based price (as discussed in Section 1) – relaxing this assumption actually strengthens our results, as discussed in Section 3.2.3.

All assumptions about the clients remain as described in section 2.1. In the absence of a credible price signal, the clients cannot distinguish between the type-h and the type-l publishers. As before, clients make their choices of publisher vs. alternative marketing service as per equations (2.1) and (2.2).

3.2.2. Separating pricing scheme for superior publishers

The focus of this subsection is on establishing the performance-based pricing scheme that can separate a publisher with a superior marketing system from one with an inferior marketing system. Figure 3.3 illustrates the relevant problem space as seen by the type-h publisher. The isoprofit line for this publisher is PP', and the isoprofit line for the type-l publisher is PQ. If the client can distinguish between superior and inferior publishers, the indifference curve of a client from the point of view of the superior publisher is IJ. This depiction is a consequence of the fact that the publisher is required to offer the competitive price as well – the client always has the alternative of purchasing marketing from this publisher at the CPM rate of p. Consequently, if the publisher successfully separates via their performance-based pricing scheme, P is still the point which anchors the indifference curve of the minimum utility level that the client requires.

Let β^h be the unconstrained optimal performance-based contract for type-h customers – this is the pricing scheme that the publishers would offer if all publishers were type-h – and by Proposition 1, it satisfies:

$$E[u(w + (1 - \beta^h)\mathbf{z}^h)] = E[u(w + \mathbf{z}^h - p)]. \tag{3.11}$$

In order for the superior publisher to signal the quality of their marketing system, the performance-based pricing scheme (α, β) needs to satisfy two conditions:

1. It must provide the client with at least as much utility as she would get if she were to choose the competitive price p from the superior publisher, and

2. The inferior publisher should get higher profits from selling at the competitive price p than from selling at (α, β).

The following proposition characterizes the optimal performance-based pricing scheme that satisfies these conditions.

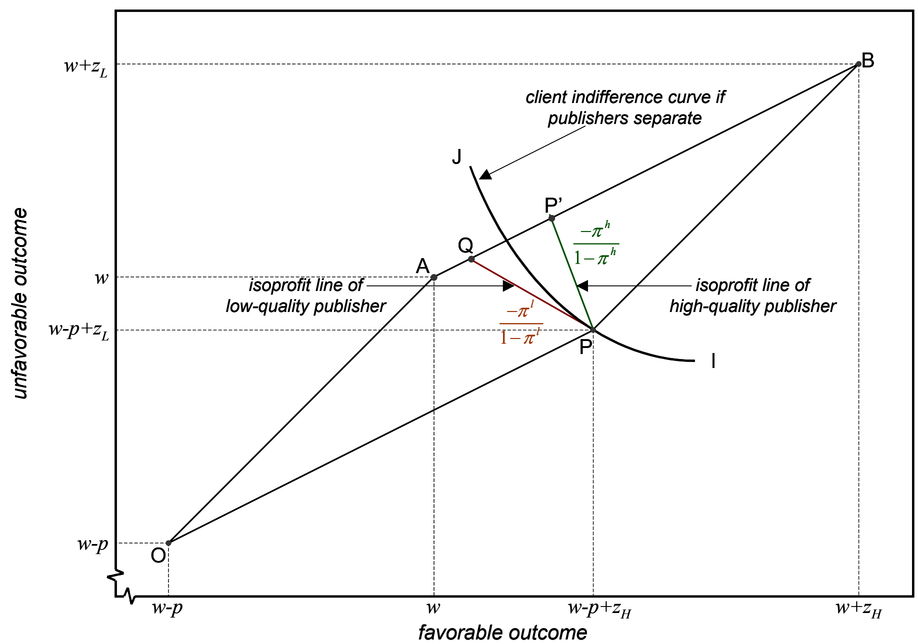

Figure 3.3: The set of feasible pricing schemes from the point of view of the type-h publisher. The contract chosen should be to the right of both the client's indifference curve IJ and the type-l publisher's isoprofit line PQ.

Proposition 7. *The optimal separating pricing scheme β^* for a type-h publisher is purely performance-based.*

(a) If $\pi^l \leq 1 - \dfrac{\beta^h z_H - p}{z_H - z_L}$, then $\beta^* = \beta^h$, and the publisher's profit per unit of marketing is

$$\Pi_{P1} = \beta^h E[\mathbf{z}^h]. \tag{3.12}$$

(b) If $\pi^l \geq 1 - \dfrac{\beta^h z_H - p}{z_H - z_L}$, then $\beta^* = \dfrac{p}{E[\mathbf{z}^l]}$, and the publisher's profit per unit of marketing is

$$\Pi_{P2} = \frac{p.E[\mathbf{z}^h]}{E[\mathbf{z}^l]}. \tag{3.13}$$

Figure 3.4 highlights the two conditions that define the publisher's choice of β^*. When the difference in effectiveness between the two publisher types is substantial, then the slope of the type-l publisher's isoprofit curve (depicted as PQ) is significantly higher (less negative) than that of the type-h publisher's isoprofit curve PP'. Consequently, the unconstrained profit-maximizing contract R for type-h, at the intersection of the indifference curve IJ and AP' is to the left of PQ and cannot be offered profitably by the type-l publisher. As a result, only the client's utility condition (condition 1) is binding, and this contract at R is the best separating pricing scheme. On the other hand, when the difference in

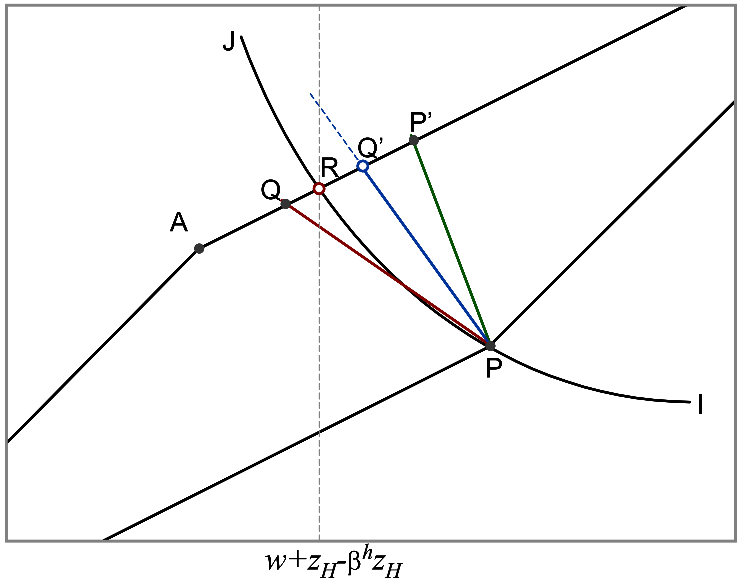

Figure 3.4: The optimal separating contracts that signal the type-h publisher's effectiveness. The type-h publisher's isoprofit line is PP'. The price is the first-best R if there is a significant gap between the types (the type-l publisher's isoprofit line is PQ, to the left of IJ). Otherwise, the type-l publisher's isoprofit line (PQ') is to the right of IJ, and the contract is at Q'.

effectiveness between the two publishers is lower, the type-l publisher's isoprofit line (depicted as PQ') has a lower (more negative) slope, and intersects the indifference curve only at P in the feasible region. Consequently, the contract at R is profitable for the type-l publisher as well, and the set of pricing schemes that separate type-h publishers is restricted to those in the area $Q'PP'$. The most profitable of these for the type-h publisher is the one at Q', and consequently, condition 2 is binding in this case.

Proposition 7 is an encouraging result, since it indicates that with a significant enough level of superiority in their marketing systems, publishers can choose first-best pricing, and there is no loss of profits due to signaling. In this case, where $\beta^* = \beta^h$, all the comparative statics and discussion results from Section 2.3.4 carry over, since the optimal pricing scheme and the profits are in fact identical to the symmetric information case. Consequently, the type-h publisher's profits are increasing in competitive price p (with a slope higher than 1), decreasing in w, decreasing for a positive shift in the support of the outcome distribution, and increasing in z_H.

On the other hand, when π^l and π^h are relatively close, the publisher needs to sacrifice profits in order to signal the superior effectiveness of their systems. In this case, the client benefits from a strict increase in utility (over what they would get from choosing the competitive price p). The comparative statics results from Section 2.3.4 are no longer applicable. However, it is evident from Proposition 7(b) that both β^* and Π_{P2} increase linearly in the competitive price p. In addition, since $\frac{E[\mathbf{z}^h]}{E[\mathbf{z}^l]} > 1$, offering a performance-based price still makes the publisher's profits more sensitive to changes in the competitive price. It is also evident that both β^* and Π_{P2} are independent of w. The following corollary establishes that the effect of changing the support of $\mathbf{z}$ is also still the same:

Corollary 2. *If* $\pi^l \geq 1 - \dfrac{\beta^h z_H - p}{z_H - z_L}$*:*

(a) An increase in the average level of marketing performance $E[\mathbf{z}]$ *via a simultaneous increase in both* z_L *and* z_H *reduces the optimal performance-based price* β^**, and also reduces the corresponding profits per unit of marketing* Π_{P2}*.*

(b)An increase in the mean and variance of marketing performance via an increase in just z_H *reduces the optimal performance-based price* β^**, but increases the profits per unit of marketing* Π_{P2}

We have assumed that the publisher has to offer the competitive price p in addition to any performance-based pricing, since this is consistent with the business reality that our model tries to capture. Relaxing this assumption does not qualitatively change the ability of the publisher to signal superior quality – in fact, it makes it easier for a high-quality publisher to separate. This is because in its absence, the client's 'reservation utility' is strictly lower – one would typically assume that the client's other option (other than the publisher's performance-based price) is to choose the alternative service (which would yield strictly lower expected utility than in our model, since expectation is taken under the belief of facing an average publisher, rather than a superior one). A stronger assumption would be that the client's alternative option is to choose the competitive price p offered by the inferior publisher. Either way, the inferior publisher's profits potential profits from imitating the superior publisher remains unchanged or reduces, and the pricing power of the superior publisher is higher.

Again, further insights based on these results are presented in Section 4.

4. Discussion and Conclusions

We conclude by providing a set of business guidelines based on our analytical results, placing our model in the context of the existing literature on contracting, and outlining current research and extensions.

4.1. Managerial insights and business guidelines

The measured performance of digital marketing continues to improve over time, due to an improved understanding and stabilizing of online consumer behavior, and also because of continuous improvements in targeting technology. As residential broadband penetration increases, bandwidth intensive rich-media marketing which garners higher responsiveness from consumers will become more widespread. Furthermore, the measures themselves are likely to become more representative of actual marketing effectiveness over time (Grahn, 2002). All of these trends suggest that understanding pricing based on measured performance is an increasingly crucial skill for publishers. Since offering a performance-based pricing option can improve publisher revenues substantially, an inability to design effective pricing schemes of this kind will place publishers at a significant competitive disadvantage.

In the absence of asymmetric information, publisher benefits are driven by their ability to bear risk more effectively. Consequently, a key prescription for publishers is that they invest in understanding the nature of the covariance of simultaneous campaign outcomes across their consumer base, or across the different Internet properties they deliver advertising to. For smaller publishers, especially those delivering email and targeted marketing to a fixed consumer base, this should include understanding the sensitivity of responsiveness to the total volume of marketing delivered to each consumer at any point in time. For larger publishers with a more stable daily volume, a sophisticated awareness of the behavior of their average portfolio of campaigns can further enhance returns from performance-based pricing. This also suggests risk–bearing-based economies of scale in the delivery of digital marketing, since larger, more diversified portfolios of campaigns can increase the performance-based pricing power of a publisher. As digital marketing becomes a progressively larger fraction of marketing budgets, this demand-side effect is likely to enhance the cost economies of scale observed by Silk and Berndt (1993).

If the actual electronic delivery of marketing is not a stand-alone product, but is offered in conjunction with higher-margin consulting or creative services (possibly as a 'loss leader' in a bundle), then using performance-based pricing for the delivery piece can increase the price flexibility that the publisher or interactive agency has on the higher-margin services. This is of specific relevance to agencies who handle their own direct marketing delivery, or larger conventional agencies who have bulk-rate contracts with Web publishers, which they then resell to their clients.

Our subsequent propositions lead to the following further guidelines:

4.1.1. Competition and revenue sensitivity

It is generally accepted that as the digital marketing industry consolidates and matures, and as consumer responsiveness improves, competitive CPM-based prices will continue to increase from their current lows. The results from Proposition 2 indicate that publishers who offer performance-based pricing will benefit disproportionately from this trend, especially if a significant portion of digital marketing continues to be sold at these CPM-based prices.

Conversely, any short-term increase in the competitive intensity in the industry affects publishers who offer performance-based pricing more adversely. This kind of increase was observed in 2001 – CPM prices fell sharply, driven partly by excess inventory and a drop in economy-wide advertising and marketing spending (but also by inflated pre-2001 pricing in general). Publishers need to be aware of the increased 'macro' variability that they face in this regard, if they offer performance-based pricing. It is important to understand that this variability is distinct from the riskiness of the outcomes – even for a publisher who is neutral to the risk of variable outcomes, the expected profits per unit of marketing themselves are more sensitive to changes in competitive CPM-based prices.

4.1.2. Client size and outcome variability

As Proposition 3 has shown, the extent to which digital marketing is a part of the client's overall marketing mix can affect the client's willingness to pay under a performance-based pricing contract. Publishers should therefore assess this attribute of their clients prior to making a performance-based pricing proposal, and offer more leveraged prices to clients who have lower overall marketing spending, and a higher fraction of digital marketing in their budgets. This bias is typical of smaller clients – some of whom may be more sensitive to payoff risk in general, due to constrained cash flows – and to those whose products are more tailored towards Internet users.

On the other hand, Proposition 4(a) indicates that a systematic increase in the industry-wide average performance of digital marketing may actually reduce the additional profits that performance-based pricing generates. If publishers have multiple distinct marketing services to price, Proposition 4(b) shows that the biggest returns from performance-based pricing are from the high-average, high-variability forms of digital marketing – those which perform well on average, but which whose 'best outcomes' are far better than the average outcome.

4.1.3. Creative quality and pricing strategy

When publishers face uncertain client creative quality, Proposition 5 indicates that designing performance-based pricing that attracts only the higher-quality clients is not possible in general. As a consequence, publishers need to make a strategic choice about the extent to which their performance-based pricing scheme will cover their client base. One strategy is *low-end coverage* – to design the scheme explicitly for their lower-quality clients (which would imply a higher variable price) . The alternative is *full coverage* – to design it to be acceptable for the clients with high-quality creative, but explicitly recognizing that this will also result in lower profits from some of their sales, to clients who have lower-quality creative.

Full coverage is viable if a significant fraction of the publisher's clients have above-average creative quality. A simple guideline for making this choice is to examine the distribution of creative quality implied by a history of outcomes across clients. If it is significantly left-skewed (that is, if a significant mass of outcomes are to the right of the median), then it suggests designing performance-based pricing to include the entire range of creative quality. It is crucial for publishers to recognize that the choice of strategy depends primarily on the relative fraction of their clients in each category – and not on absolute quality levels. In addition, it is always a sub-optimal strategy to take a middle-ground approach, and design a performance-based pricing scheme attractive to the client with 'average' creative quality. This will simply result in the same adoption outcome as the low-end coverage strategy, but with lower average profits per client.

4.1.4. Communicating technological effectiveness

Proposition 7 shows that a performance-based pricing scheme can be an effective signal of superior technology, of a more effective format, or of a higher-quality target consumer base. Moreover, when a publisher believes that their effectiveness is substantially higher than that of the average provider, then the consideration of separating from the competition involves no additional pricing complexity – simply offering the globally optimal performance-based pricing scheme, along with the competitive CPM-based rate, will automatically signal their superior effectiveness. Ease of implementation is critical from point of view of business execution, and this result suggests that signaling using performance-based pricing is actually likely to be implemented. This is a reassuring finding, since the ability to easily signal technological superiority is critical for the sustained improvement of marketing delivery technologies.

If the differences in effectiveness across publishers is not very significant, then Proposition 7(b) indicates that communicating superiority, while theoretically possible, is contingent on a clear understanding of the quality of competing services. This is often hard to assess precisely in practice. Consequently, the result suggests that when a publisher does not have a significant technological lead, the objective of signaling quality may be hard to implement.

4.2. Relationship to the screening and signaling literature

Our representation of competition through an exogenous outside good differs from the standard approach in the literature (which traditionally considers either a monopoly or perfectly competitive markets with zero-profit equilibria), and has been specifically chosen to represent our business context most appropriately. It may apply to other industries as well. While some of our sensitivity results (such as the wealth effects) parallel those from the related literature, most others, such as the impact of varying the exogenous price and the outcome distribution's support, are different. The fact that the symmetric information comparative statics generalize to many of the asymmetric information models is also significant.

Our model of screening clients with unknown characteristics can be benchmarked with related models from other industries. For instance, the seminal paper by Rothschild and Stiglitz (1976) examined screening insurance clients with different risk profiles, and more recent work by Rubinfield and Scotchmer (1993) suggests that contingency fees may address the problem lawyers face when screening clients with uncertain case quality. Our results are similar to the extent that the contract for the 'lower' customer type provides more insurance against risk, and the higher customer type tends not to get any surplus over their 'reservation level'. Since we depart from these standard models in our representation of competition, and in the constraints that we place on the seller's number of and parameters for contracts, our equilibrium performance-based contracts are profitable, though they do not involve either full-insurance or separation of client types in equilibrium . We could extend our model to allow multiple performance-based contracts, to benchmark further, and this is something we discuss in Section 4.3.

Our model of signaling technological quality falls under the class of problems with an informed principal (rather than an informed agent, as in Spence, 1973, and the literature that followed) under common values (see Maskin and Tirole, 1992), and in which the contract itself is the only signal available. Similar work includes Leland and Pyle (1977), who demonstrate how an entrepreneur can signal firm or project quality through the fraction of risky equity that she retains; Gallini and Wright (1990), who have shown that output-based royalties can signal technology quality in optimal licensing contracts between an informed patent holder and an uninformed licensee; and Desai and Srinivasan (1995), who derive contracts under which an informed franchisor can use service levels to signal demand potential to an uninformed franchisee. We establish the feasibility of always successfully signaling quality even when the seller is required to offer the competitive non-performance-based price – the requirement is a more 'stringent' condition than is normally imposed. Under this constraint, the result that our seller can in fact achieve first-best payoffs in a fraction of cases is new. Again, the analysis of how price and profitability varies with competition, risk and client size is a feature which differentiates our work from standard signaling models.

4.3. Ongoing research

Our ongoing research is focused on relaxing the restriction of a single performance-based pricing scheme. Realistically, digital marketing firms, who submit competitive proposals to get their business, often do not have the liberty of suggesting multiple pricing options, especially when their clients prefer simple, easy-to-benchmark pricing schemes. On the other hand, from a theoretical point of view, the problem merits further analysis. Preliminary results indicate no substantial changes in the results of Section 3 – this remains work in progress, however. We are also working on extending some of the results of Section 3 to include bilateral information asymmetry (simultaneous client and publisher quality uncertainty).

Another area of our current research aims to capture specific aspects of the sales process as variables in designing the pricing schemes. Most publishers have standard rate cards for their CPM-based and performance-based pricing schemes. However, the final price that the client pays is almost always lower – reductions of 25% to 50% are the norm. The sales force typically has some control over the extent to which they can reduce the official price, though setting these guidelines optimally, in the presence of commission-based compensation for the salespeople, is a significant challenge for most publishers. A related problem has been addressed by Lal (1986). We hope to address these extensions in the near future.

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A. Appendix: Proofs

Proof of Lemma 1

In Figure 1(b), the client's indifference curve at a constant level of utility U is governed by the following equation:

$$\pi u(x) + (1-\pi)u(y) = U. \tag{A.1}$$

Differentiating both sides of (A.1) with respect to x yields

$$\pi u'(x) + (1-\pi)u'(y)\frac{dy}{dx} = 0, \tag{A.2}$$

which implies that the slope of the client's indifference curve at any point (x, y) in the space is

$$\frac{dy}{dx} = \frac{-\pi u'(x)}{(1-\pi)u'(y)}. \tag{A.3}$$

Now, the publisher's profit at the point P is p. The isoprofit line through P is governed by the equation:

$$\pi(z_H - x) + (1-\pi)(z_L - y) = p, \tag{A.4}$$

which reduces to

$$y = \left(\frac{\pi z_H + (1-\pi)z_L}{(1-\pi)}\right) - \left(\frac{\pi}{1-\pi}\right)x. \tag{A.5}$$

The slope of the isoprofit line PP' is therefore $\frac{-\pi}{1-\pi}$. Also, the region in which publisher profits are strictly higher than p is therefore

$$y < \left(\frac{\pi z_H + (1-\pi)z_L}{(1-\pi)}\right) - \left(\frac{\pi}{1-\pi}\right)x, \tag{A.6}$$

which is the entire region to the left of and below the line specified by (A.5). Note that the portion of the feasible set $OABP$ that lies in this region is completely below the 45° line, or every point in this region satisfies $x > y$. Since $u'(.)$ is strictly decreasing, this means that in the portion of the indifference curve IJ through P that lies in this area, $u'(x) < u'(y)$, which when combined with (A.3) implies that the slope of the indifference curve is strictly higher (less negative) than the slope of the isoprofit line in this region, which means that $KP'P$ has strictly positive area. The result follows.

Proof of Proposition 1

It is easily verified that the pricing scheme specified by $\alpha^* = 0$, and

$$E[u(w + (1 - \beta^*)\mathbf{z})] = E[u(w + \mathbf{z} - p)] \tag{A.7}$$

is feasible, and that any contract in which $\beta > \beta^*$ reduces client utility below $E[u(w+\mathbf{z}-p)]$. Assume that there is another pricing scheme $(\alpha, \beta^* - \varepsilon)$ which improves publisher profits, that is:

$$\alpha + E[(\beta^* - \varepsilon)\mathbf{z}] \geq E[\beta^* \mathbf{z}] \tag{A.8}$$

and that this pricing scheme will be adopted by the client, that is:

$$E[u(w + (1 - (\beta^* - \varepsilon))\mathbf{z} - \alpha)] \geq E[u(w + \mathbf{z} - p)]. \tag{A.9}$$

(A.7) and (A.9) imply that

$$E[u(w + (1 - (\beta^* - \varepsilon))\mathbf{z} - \alpha)] \geq E[u(w + (1 - \beta^*)\mathbf{z})]. \tag{A.10}$$

Denote the expectation of $\mathbf{z}$ as $\bar{z}$. Adding and subtracting $\varepsilon\bar{z}$ to the LHS of (A.10), and rearranging terms inside the parenthesis yields

$$E[u(w + (1 - \beta^*)\mathbf{z} + \varepsilon(\mathbf{z} - \bar{z}) + (\varepsilon\bar{z} - \alpha)] \geq E[u(w + (1 - \beta^*)\mathbf{z})]. \tag{A.11}$$

Since $E[\varepsilon(\mathbf{z} - \bar{z})] = 0$, and $u(.)$ is strictly concave, we know that

$$E[u(w + (1 - \beta^*)\mathbf{z} + \varepsilon(\mathbf{z} - \bar{z})] < E[u(w + (1 - \beta^*)\mathbf{z})]. \tag{A.12}$$

Therefore, for (A.11) and (A.12) to be simultaneously true, it must be the case that

$$(\varepsilon\bar{z} - \alpha) > \mathbf{0}. \tag{A.13}$$

Now, the publisher's expected profits from $(\alpha, \beta^* - \varepsilon)$ an be rewritten as

$$\alpha + E[(\beta^* - \varepsilon)\mathbf{z}] = \beta^* E[\mathbf{z}] - (\varepsilon\bar{z} - \alpha). \tag{A.14}$$

Consequently, (A.13) and (A.14) together imply that

$$\alpha + E[(\beta^* - \varepsilon)\mathbf{z}] < \beta^* E[z],$$

which contradicts (A.8). This establishes that β^* is optimal. Since (A.8) is a weak inequality, this contraction also establishes uniqueness, which completes the proof.

Proof of Proposition 2

From Proposition 1, β^* is defined by:

$$E[u(w + (1-\beta^*)\mathbf{z})] = E[u(w + \mathbf{z} - p)]. \tag{A.15}$$

Totally differentiating both sides of (A.15) with respect to p yields:

$$-\frac{\partial \beta^*}{\partial p} E[\mathbf{z}u'(w + (1-\beta^*)\mathbf{z})] = -E[u'(w + \mathbf{z} - p)], \tag{A.16}$$

which implies that

$$\frac{\partial \beta^*}{\partial p} = \frac{E[u'(w + \mathbf{z} - p)]}{E[\mathbf{z}u'(w + (1-\beta^*)\mathbf{z})]}. \tag{A.17}$$

Since $\mathbf{z}$ has strictly positive support, and $u'(x) > 0$, (A.17) implies that $\frac{\partial \beta^*}{\partial p} > 0$.

Now, since $\frac{\pi z_H}{E[\mathbf{z}]} > \pi$, and $u'(.)$ is strictly decreasing, it is easily shown that

$$\pi u'(w + z_H - \beta^* z_H) + (1-\pi)u'(w + z_L - \beta^* z_L) \geq \frac{\pi z_H}{E[\mathbf{z}]} u'(w + z_H - \beta^* z_H) + \frac{(1-\pi)z_L}{E[\mathbf{z}]} u'(w + z_L - \beta^* z_L),$$

which implies that

$$E[u'(w + (1-\beta^*)\mathbf{z})].E[\mathbf{z}] \geq E[\mathbf{z}u'(w + (1-\beta^*)\mathbf{z})]. \tag{A.18}$$

Equations (A.17) and (A.18) together imply that

$$\frac{\partial \beta^*}{\partial p} \geq \frac{1}{E[\mathbf{z}]} \frac{E[u'(w + \mathbf{z} - p)]}{E[u'(w + (1-\beta^*)\mathbf{z})]}. \tag{A.19}$$

Now, since $p \leq \beta^* E[\mathbf{z}]$ and $u'(.)$ is strictly decreasing, we know that

$$E[u'(w + \mathbf{z} - p)] \geq E[u'(w + \mathbf{z} - \beta^* E[\mathbf{z}])]. \tag{A.20}$$

For any $\beta^* > 0$, the random variable $w + (\mathbf{z} - \beta^* E[\mathbf{z}])$ is a mean-preserving spread of the random variable $w + \mathbf{z}(\mathbf{1} - \beta^*)$. Consequently, the convexity of $u'(.)$ implies that

$$E[u'(w + \mathbf{z} - \beta^* E[\mathbf{z}])] > E[u'(w + (1-\beta^*)\mathbf{z})]. \tag{A.21}$$

Equations (A.20) and (A.21) together imply that

$$E[u'(w + \mathbf{z} - p)] > E[u'(w + (1-\beta^*)\mathbf{z})], \tag{A.22}$$

which when combined with (A.19) yields

$$\frac{\partial \beta^*}{\partial p} > \frac{1}{E[\mathbf{z}]}, \tag{A.23}$$

or that $\frac{\partial \beta^*}{\partial p} E[\mathbf{z}] > 1$, which completes the proof.

Proof of Proposition 3

Totally differentiating both sides of (A.15) with respect to w yields:

$$\frac{\partial \beta^*}{\partial w} = \frac{E[u'(w+(1-\beta^*)\mathbf{z})] - E[u'(w+\mathbf{z}-p)]}{E[\mathbf{z}u'(w+(1-\beta^*)\mathbf{z})]}. \tag{A.24}$$

Equations (A.22) and (A.24) together imply that $\frac{\partial \beta^*}{\partial w} < 0$. This establishes the result.

Proof of Proposition 4(a)

Totally differentiating both sides of (A.15) with respect to $\mathbf{z}$ yields

$$\frac{\partial \beta^*}{\partial \mathbf{z}} = \frac{(1-\beta^*)E[u'(w+(1-\beta^*)\mathbf{z})] - E[u'(w+\mathbf{z}-p)]}{E[\mathbf{z}u'(w+(1-\beta^*)\mathbf{z})]}, \tag{A.25}$$

where a change in $\mathbf{z}$ should be interpreted as a corresponding identical change in all elements of the support of $\mathbf{z}$. We know from equation (A.20) that

$$E[u'(w+\mathbf{z}-p)] \geq E[u'(w+\mathbf{z}-\beta^* E[\mathbf{z}])], \tag{A.26}$$

which when combined with (A.25) yields $\frac{\partial \beta^*}{\partial \mathbf{z}} < 0$. The corresponding change in publisher profits is:

$$\frac{\partial}{\partial \mathbf{z}}\beta^* E[\mathbf{z}] = \beta^* + \frac{\partial \beta^*}{\partial \mathbf{z}} E[\mathbf{z}]. \tag{A.27}$$

Substituting equation (A.25) into equation (A.27) and rearranging, we get:

$$\begin{aligned}\frac{\partial}{\partial \mathbf{z}}\beta^* E[\mathbf{z}] &= E[\mathbf{z}].\frac{E[u'(w+(1-\beta^*)\mathbf{z})] - E[u'(w+\mathbf{z}-p)]}{E[\mathbf{z}u'(w+(1-\beta^*)\mathbf{z})]} + \\ & \quad \beta^*\left(1 - \frac{E[\mathbf{z}].E[u'(w+(1-\beta^*)\mathbf{z})]}{E[\mathbf{z}u'(w+(1-\beta^*)\mathbf{z})]}\right).\end{aligned} \tag{A.28}$$

Equation (A.25) implies that the first term on the RHS of equation (A.28) is negative, and equation (A.18) implies that the second term is non-positive. The result follows.

Proof of Proposition 4(b)

Totally differentiating both sides of (A.15) with respect to z_H, and rearranging terms yields:

$$\frac{\partial \beta^*}{\partial z_H} = \pi \frac{u'(w+(1-\beta^*)z_H) - u'(w+z_H-p) - \beta^* u'(w+(1-\beta^*)z_H)}{E[\mathbf{z}u'(w+(1-\beta^*)\mathbf{z})]}. \tag{A.29}$$

Since $\beta^* z_H > p$, and $u'(.)$ is strictly decreasing, we know that

$$u'(w+(1-\beta^*)z_H) > u'(w+z_H-p). \tag{A.30}$$

Now,

$$\frac{\partial}{\partial z_H}\beta^* E[\mathbf{z}] = \pi\beta^* + \frac{\partial \beta^*}{\partial z_H} E[\mathbf{z}]. \tag{A.31}$$

Substituting equation (A.29) into equation (A.31) and rearranging terms yields:

$$\begin{aligned} \frac{\partial}{\partial z_H}\beta^* E[\mathbf{z}] &= \pi E[\mathbf{z}]\frac{u'(w+(1-\beta^*)z_H) - u'(w+z_H-p)}{E[\mathbf{z}u'(w+(1-\beta^*)\mathbf{z})]} + \\ &\quad \pi\beta^*\left(1 - \frac{u'(w+(1-\beta^*)z_H}{\frac{1}{E[\mathbf{z}]}E[\mathbf{z}u'(w+(1-\beta^*)\mathbf{z})]}\right). \end{aligned} \tag{A.32}$$

From equation (A.30), the first term on the RHS of equation (A.32) is positive. Moreover, since

$$\frac{1}{E[\mathbf{z}]}E[\mathbf{z}u'(w+(1-\beta^*)\mathbf{z})] = \frac{\pi z_H}{E[\mathbf{z}]}u'(w+z_H-\beta^* z_H) + \frac{(1-\pi)z_L}{E[\mathbf{z}]}u'(w+z_L-\beta^* z_L), \tag{A.33}$$

which is a convex combination of $u'(w+(1-\beta^*)z_H)$ and $u'(w+(1-\beta^*)z_L)$. Since $u'(.)$ is strictly decreasing, this implies that

$$u'(w+(1-\beta^*)z_H > \frac{1}{E[\mathbf{z}]}E[\mathbf{z}u'(w+(1-\beta^*)\mathbf{z})], \tag{A.34}$$

and consequently, the second term on the RHS of equation (A.32) is also positive, which completes the proof.

Proof of Proposition 5

The proof is based on the following lemma:

Lemma 2. *For any $\pi > 0$, if the contract (α, β) is simultaneously weakly profit improving for the publisher and weakly preferred by a client over the competitive price p, then $\beta z_H + \alpha > p > \beta z_L + \alpha$*

Proof: Assume that $p \geq \beta z_H + \alpha$. This implies that $p > \beta z_L + \alpha$, which in turn implies that $p > \alpha + \beta(\pi z_H + (1-\pi)z_L)$, and consequently, the contract (α, β) is not profit improving for the publisher. Similarly, if $p \leq \beta z_L + \alpha$, which implies that $p < \beta z_H + \alpha$. These in turn imply that $u(w + z_L - p) \geq u(w + (1-\beta)z_L)$ and $u(w + z_H - p) > u(w + (1-\beta)z_H)$, which implies that the competitive price p is strictly preferred by the client over the contract (α, β). The result follows.

Now, assume the converse of Proposition 5, that is, there exists a contract (α, β) such that

$$E[u(w+(1-\beta)\mathbf{z}^h - \alpha)] \geq E[u(w+\mathbf{z}^h - p)], \tag{A.35}$$

and

$$E[u(w+(1-\beta)\mathbf{z}^l - \alpha)] \leq E[u(w+\mathbf{z}^l - p)]. \tag{A.36}$$

Subtracting equation (A.36) from (A.35):

$$E[u(w+(1-\beta)\mathbf{z}^h - \alpha)] - E[u(w+(1-\beta)\mathbf{z}^l - \alpha)] \geq E[u(w+\mathbf{z}^h - p)] - E[u(w+\mathbf{z}^l - p)], \tag{A.37}$$

which upon expanding and simplifying yields:

$$(\pi^h - \pi^l)[u(w+(1-\beta)z_H - \alpha) - u(w+(1-\beta)z_L - \alpha)] \geq (\pi^h - \pi^l)[u(w+z_H-p) - u(w+z_L-p)]. \quad \text{(A.38)}$$

However, Lemma 2 implies that $w + z_H - (\beta z_H + \alpha) < w + z_H - p$ and $w + z_L - (\beta z_L + \alpha) > w + z_H - p$, or that

$$w + (1-\beta)z_H - \alpha < w + z_H - p \quad \text{(A.39)}$$

and

$$w + (1-\beta)z_L - \alpha > w + z_H - p. \quad \text{(A.40)}$$

Since $u(.)$ is strictly increasing, equations (A.39) and (A.40) together imply that

$$u(w + (1-\beta)z_H - \alpha) - u(w + (1-\beta)z_L - \alpha) < u(w + z_H - p) - u(w + z_L - p), \quad \text{(A.41)}$$

which contradicts (A.38) for any $\pi^h \geq \pi^l$. The result follows.

Proof of Proposition 6

From Proposition 5, we know that any contract (α, β) that is adopted by type-h customers will also be adopted by type-l customers. If the publisher chooses to offer a contract adopted only by type-l customers, then by Proposition 1, the most profitable one is β^l. On the other hand, suppose the publisher chooses to offer a contract adopted by both customer types. Based on Proposition 5, and the definition of β^h, we know that β^h will be adopted by both customer types. Moreover, from the proof of Proposition 1, we know that β^h is strictly more profitable than any contract (α, β) adopted by all customers for which $\alpha > 0$. Since (3.4) is a strict equality, any increase in β^h causes the type-h customers to switch to the CPM-based price p. Any reduction in β^h strictly reduces profits. Therefore, if the seller chooses to offer a contract adopted by both types, β^h is the appropriate one. The result follows by computing the profits under β^l:

$$\Pi_{C1} = \lambda p + (1-\lambda)\beta^l E[\mathbf{z}^l], \quad \text{(A.42)}$$

and under β^h:

$$\Pi_{C2} = \beta^h(\lambda E[\mathbf{z}^h] + (1-\lambda)E[\mathbf{z}^l]), \quad \text{(A.43)}$$

and specifying the range of λ for which $\Pi_{C1} > (<)\ \Pi_{C2}$ by comparing (A.42) and (A.43).