Product scope and bilateral entry deterrence in converging technology industries

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Abstract: We model the bilateral threat of entry between firms in two industries, which is characteristic of a number of IT industries. An endogenous choice of product scope by firms in each industry affects both the extent of product differentiation for incumbent oligopolists as well as the fixed costs of a potential entrant. Our analysis establishes unique symmetric equilibrium choices of scope and price, both in the absence and the presence of an entry threat. When entry is threatened bilaterally, in equilibrium, firms may symmetrically either deter entry into their core industry, or accomodate it while entering the neighboring industry. Even in the absence of technological shocks, we show how steady progress in technology can result in switching between these equilibria, leading to the periodic and sudden shifts in industry concentration and firm profitablity; such shifts have been observed during the "competitive crash" in the computer industry, and more recently, on account of digital convergence.

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1. Overview and motivation

This paper studies a kind of bilateral entry threat that is prevalent in a number of pairs of information technology industries. Our motivation is best illustrated by some examples. Progress in networking and communications technology, along with the ability to digitize and transmit voice over IP data networks has made it feasible for the providers of cable television to enter the residential voice telephony market. Simultaneously, this technology also makes it feasible for telephone companies to become providers of digital video entertainment over their telephone lines, which has led them to begin threatening entry into the core business of cable television providers. The viability of this kind of bilateral entry threat increases with technological progress that leads to higher-bandwidth data networks, as well as more sophisticated compression algorithms. Similarly, a move towards operating system and general-purpose hardware based product architectures in both the cellular handset (Symbian OS, Smartphone) and handheld computing (PalmOS, PocketPC) markets has led to analogous bilateral entry of firms into each others' markets. Progress in semiconductor technology makes the underlying hardware more powerful in each industry, leading to a corresponding increase in the viability of this kind of bilateral entry.

This kind of technology convergence across industries presents a new set of strategic considerations for incumbent firms: deciding whether to enter a rival market while simultaneously considering the effects of their related product and pricing choices on entry deterrence and oligopoly profits within their own market. Critical to this choice is *product scope*, which affects product value, the extent of product differentiation, and the fixed costs of entry. While technological progress makes this kind of bilateral entry more technologically viable, it is not immediately clear whether it is profitable, or will emerge as equilibrium behavior by strategic firms.

We begin by analyzing a model of imperfect competition in which the choice of both price and product scope is endogenous. The latter reflects the fact that in many information technology-based industries, firms have substantial control over the scope, or the breadth of functionality provided by their products. Developers of enterprise software suites and operating systems have broad control over the set of diverse features and modules that can be integrated into their packages. Manufacturers of general purpose computing chips make explicit design choices about the level of integrated cache, degree of power consumption, multimedia and graphical support provided in their chips, thereby affecting the chips' performance on a variety of applications, and consequently, the scope of devices these chips can be used in. Manufacturers of networking switches can decide whether or not to include routing capabilities in their equipment, and if they do, also choose the type of routing and protocols they support.

The ability to choose scope endogenously has many effects on demand and profitability. Firstly, it provides firms with an additional degree of freedom in increasing the value of their products. Rather than leading to a simple increase in quality (which would suggest a model of vertical differentiation), or an emphasis on one set of features at the cost of others (characteristic of the horizontal differentiation models that follow Hotelling, 1927 and Salop, 1979), an increase in scope increases value for the product for a broad set of horizontally differentiated customers.

Second, increasing the scope of a product also typically increases its fixed costs while leaving variable costs relatively unchanged (this is especially true in the software and microprocessor industries). An aspect of Microsoft's strategy that has received somewhat less attention than their aggressive entry into the browser market is that they have increased the product scope of their core Windows operating system to the point where the fixed costs of entry are substantial (upto $9 billion, according to Hall and Hall, 2000). This highlights the role that costly and endogenously chosen product scope can play in deterring entry.

Thirdly, the extent to which an increase in product scope increases fixed costs is influenced by progress in information technology – sometimes within the technology market, but also commonly in an upstream supplier industry, or in a downstream industry for a complementary product (Economides and Salop, 1992). This has been recognized in models of general-purpose technologies (for instance, Breshanan and Trajtenberg, 1995). Steady progress in the upstream semiconductor industry continually relaxes constraints on electronics product design, allowing firms in the computer and home electronics industries to achieve increases in scope at a lower fixed cost. The same logic holds for desktop software developers, whose design and architecture costs are driven by how powerful the machines (supplied by the downstream PC industry) their software will run on are. Additionally, development of new methodologies within the software industry, such as object oriented technologies and software engineering, along with new tools to aid the design process also reduce the fixed costs of increasing product scope.

Together, these factors indicate that in information technology industries, two key factors that determine equilibrium pricing and industry concentration in standard models – the intensity of imperfect competition and the level of fixed costs – are no longer exogenous. In addition, the fixed cost function itself changes rapidly over time, as technology progresses. Section 3 presents our model of symmetric oligopoly that incorporates each of these features. We derive the equilibrium sequential choices of product scope and price. In the absence of an entry threat, there is a unique symmetric subgame perfect Nash equilibrium. We characterize how strategic choices and surplus vary with changes in industry concentration and technological progress: the flexibility of product scope is shown to moderate (rather than intensify) the effect of an increase in industry concentration on the extent of price competition, and higher industry concentration is not necessarily socially beneficial, though it always increases consumer surplus. We subsequently derive the unique equilibrium choices of product scope and pricing that deter potential entry. Not surprisingly, higher product scope is necessary to deter entry, though this increase may reduce total surplus. Our analysis is more involved than existing models of oligopoly entry deterrence (Gilbert and Vives, 1986), since competing firms also control the level of fixed costs that influence the entry decision, through their choice of product scope.

The variation in equilibrium firm behavior induced by an entry threat forms the basis for our model of bilateral entry, presented in Section 4. Firms in a pair of "adjacent" oligopoly markets of the kind described above each decide whether or not to accommodate or deter entry through their choice of product scope, and also choose whether or not to enter the adjacent industry. We establish that there are two possible equilibria: one in which firms in each industry choose lower product scope, enter the adjacent industry, and accommodate entry in their own, and another in which firms in each industry choose higher product scope, do not enter the adjacent industry and successfully deter entry into their own.

Exactly one of these two outcomes is an equilibrium for a given pair of industries. When the profits from successfully deterring entry are lower than the profits from one's *own* industry under accommodation (this is half the total profits under the accommodate-enter equilibrium), the former equilibrium is the unique outcome. The equilibrium in which all firms accommodate and enter is Pareto-efficient; the one under which all firms deter entry is often not.

Based on these equilibria, we discuss how technological progress that varies the relative magnitude of payoffs under each strategy can lead to discontinuous shifts in industry concentration and incumbent profits, as the equilibrium switches from bilateral deterrence to bilateral accommodation, and vice versa. One of our observations in this regard is that these sudden changes need not be preceded by a technological shock, but can emerge as a natural strategic response to gradual technological progress. The "competitive crash" in the computer industry in the early 1990's, and the current industry realignment induced by digital convergence are good illustrations, since both were consequences of gradual (though rapid) technological progress, rather than any kind of sudden breakthrough innovation. Our model indicates that the shift may lead to either a sudden increase in concentration in both industries, lower product scope, higher firm profits and lower consumer surplus, or to a diametrically opposite effect: increases in product scope, firms receding into their core industries which become more concentrated, lower profits and higher consumer surplus.

The rest of this paper is organized as follows. Section 2 introduces our underlying model of oligopoly with endogenous product scope and entry costs, and Section 3 derives its equilibria with and without the threat of entry. Section 4 characterizes the equilibria for a game of bilateral entry across two industries. Section 5 concludes and suggests directions for future work.

2. Overview of model

This section outlines our oligopoly model with endogenous and costly choice of product scope, which builds on earlier models of monopolistic competition by von Ungern-Sternberg (1988) and Hendel and Figueiredo (1997).

2.1. Firms and products

Each potential product is represented by a point on the unit circle. There are n firms, each of which produces exactly one product, and who share identical production technology. Following the prior literature (for instance, Economides, 1989), firms are symmetrically located around the unit circle. Each firm j makes a costly choice $s_j \in (0, \infty)$ of *product scope*, and a choice of price p_j. For analytical simplicity, we assume a constant unit variable cost of production c. The fixed cost of scope depends on an exogenous *state of technology* τ and on the level of scope s_j chosen. We make the following assumptions about the fixed cost function $F(s, \tau)$.

1. $F(s, \tau) > 0, F_1(s, \tau) > 0, F_{11}(s, \tau) > 0$: Fixed costs are positive, increasing and convex in

scope.

2. $F_2(s,\tau) < 0, F_{22}(s,\tau) > 0$: The cost of providing a fixed level of scope is decreasing and convex in the state of technology τ.

3. $F_{12}(s,\tau) < 0$: The fixed cost of every unit increase in scope is lower at higher states of technology.

Numbered subscripts of functions represent partial derivatives with respect to the corresponding variable. This notation is preserved throughout the paper. The first assumption posits fixed costs that are convex in scope, which is characteristic of many information technology products. For example, the fixed cost of developing software is convex in the number of lines of code, and in the number of function points, both of which increase as one expands the scope of software functionality. Alternately, if the number of lines of code is constrained by design guidelines based on average end-user memory or processor constraints, then adding each new functionality requires a increasing level of investment in careful software architecture and optimization. Similarly, design costs for electronic devices increase at an increasing rate if engineers have to incorporate increasing functionality onto a circuit board of limited dimensions, with constraints on total battery needs and heat emission.

The next assumption describes how these fixed costs change with technological progress. For instance, the fixed cost of delivering a specified level of functionality in a semiconductor-based device like a cellular handset decreases continuously as the raw power of semiconductor technology increases. This decrease in fixed costs is not due to a drop in the price of chips[1]. Rather, more powerful microprocessors, DSP chips, memory chips and higher feasible levels of miniaturization induce a lower investment requirement in product design or software architecture to deliver the same level of functionality in a device. Analogously, when users have personal computers with faster CPUs and more RAM, manufacturers of software can increase the size of their code base with a lower performance impact, and therefore, can increase product scope with less careful software architecture and optimization.

[1] Note that there may also be a decrease in the variable cost of production due to a decrease in the cost of chips. This effect is likely to strengthen our results; however, in this paper, we focus on changes in fixed costs.

2.2. Consumers

There is a mass of customers of total size m distributed uniformly around the circle. The preferences of a customer located at distance x_j from firm j's product are represented by the utility function $U(x_j, s_j)$, where

$$U(x_j, s_j) = v - x_j t(s_j).$$

$t(.)$ is the misfit cost function that relates product scope to misfit or transportation costs. This function is assumed to have the following properties:

1. $t(s_j) > 0, t_1(s_j) < 0$: Unit cost of misfit is positive and decreasing in scope
2. $t_{11}(s_j) > 0, \frac{\partial}{\partial s_j}(\frac{-t_1(s_j)}{(t(s_j))^2}) \leq 0$: The unit cost of misfit is sufficiently convex in scope

The shape of $t(s)$ reflects products becoming more general-purpose and being able to satisfy functionality requirements of a broader set of consumers better as their scope increases However, progressive incremental benefits from increases in scope are generally diminishing. Assuming that $t(s_j)$ is convex is necessary to ensure well-behaved best-response functions[2].

Each consumer purchases exactly one product: the one that maximizes their surplus $[U(x_j, s_j) - p_j]$. As is customary in location models of product differentiation, we assume that v is high enough so that all consumers get non-zero surplus from at least one product in equilibrium. The sequence of events is specified independently in each subsequent section.

3. Symmetric oligopoly with and without an entry threat

This section analyzes equilibrium sequential product scope and pricing choices for the oligopoly model outlined in Section 2, first in the absence of an entry threat, and next, when the oligopolists. collectively make choices that deter a threat of entry. These results extend existing work on monopolistic competition with endogenous scope (von Ungern-Sternberg, 1988) and on oligopolistic entry deterrence (Gilbert and Vives, 1986). They also form the basis for the model of bilateral entry deterrence which is presented in Section 4.

[2] The condition is equivalent to assuming that $2(t_1(s))^2 - t(s)t_{11}(s) \leq 0$. It is satisfied, for instance, for any mixture of polynomials $\sum_{i=1}^{n} a_i s^{-b_i}$ in which $a_i \geq 0, b_i \geq 1$.

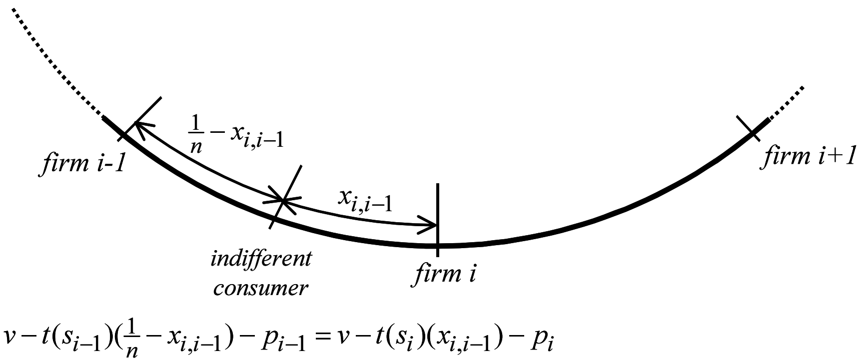

Figure 3.1: Illustrates the basis for thedemand functions faced by each competing firm.

3.1. Consumer choice and demand

If firm i chooses price and scope (p_i, s_i), and firm $i-1$ chooses price and scope (p_{i-1}, s_{i-1}), a consumer located between these firms chooses product i if

$$v - p_i - xt(s_i) \geq v - p_{i-1} - (\frac{1}{n} - x)t(s_{i-1}). \tag{3.1}$$

The consumer indifferent between firm i and firm $i-1$ is therefore located at $x_{i,i-1}$, where:

$$v - p_i - x_{i,i-1}t(s_i) = v - p_{i-1} - (\frac{1}{n} - x_{i,i-1})t(s_{i-1}). \tag{3.2}$$

Refer to Figure 3.1. Therefore,

$$x_{i,i-1} = \frac{p_{i-1} - p_i + \frac{1}{n}t(s_{i-1})}{t(s_i) + t(s_{i-1})}. \tag{3.3}$$

The demand received by firm i from the segment between i and $i-1$ is $mx_{i,i-1}$. Consequently, it follows that if firm i chooses (p_i, s_i), and its two adjacent neighbors firm $i-1$ and firm $i+1$ choose (p_{i-1}, s_{i-1}), and (p_{i+1}, s_{i+1}), then the demand for firm i's product is:

$$q(p_i, s_i | p_{i-1}, s_{i-1}, p_{i+1}, s_{i+1}) = m\left(\frac{p_{i-1} - p_i + \frac{1}{n}t(s_{i-1})}{t(s_i) + t(s_{i-1})} + \frac{p_{i+1} - p_i + \frac{1}{n}t(s_{i+1})}{t(s_i) + t(s_{i+1})}\right). \tag{3.4}$$

Define π as the *gross* profit function and Π as the *net* profit function (gross profits less fixed costs of scope). These functions are:

$$\pi(p_i, s_i|p_{i-1}, s_{i-1}, p_{i+1}, s_{i+1}) = (p_i - c)q(p_i, s_i|p_{i-1}, s_{i-1}, p_{i+1}, s_{i+1}); \tag{3.5}$$

$$\Pi(p_i, s_i|p_{i-1}, s_{i-1}, p_{i+1}, s_{i+1}) = \pi(p_i, s_i|p_{i-1}, s_{i-1}, p_{i+1}, s_{i+1}) - F(s_i, \tau), \tag{3.6}$$

and form the basis for the payoff functions in different stages of the games that follow.

3.2. Oligopoly: no threat of entry

The sequence of events in our first game is as follows: In stage 1, the n firms simultaneously choose their levels of product scope. In stage 2, with complete information about the stage 1 choices, the n firms simultaneously choose prices. Finally, based on the product offerings and prices, consumers make their purchase decisions, and the firms receive their payoffs. We focus our attention on symmetric subgame perfect equilibria of the game. The unique such equilibrium is specified in Proposition 1. All proofs are in Appendix A.

Proposition 1. *For an n-firm oligopoly with no threat of entry, the symmetric sub-game perfect Nash equilibrium choice of price and scope, denoted $p^*_A(n), s^*_A(n)$, are specified by:*

$$F_1(s^*_A(n), \tau) = \frac{-mt_1(s^*_A(n))}{4n^2}; \tag{3.7}$$

$$p^*_A(n) = c + \frac{t(s^*_A(n))}{n}. \tag{3.8}$$

The choice of the subscript (and subsequently, the superscript) A is because these functions are later used to derive payoffs and welfare under entry *accommodation*, in contrast to entry *deterrence*.

Under the equilibrium of Proposition 1, each firm's profits are:

$$\Pi^A(s^*_A(n), n) = \frac{mt(s^*_A(n))}{n^2} - F(s^*_A(n), \tau), \tag{3.9}$$

consumer surplus is:

$$C^A(s^*_A(n), n) = m\left[2n\int_0^{\frac{1}{2n}}\left(v - xt(s^*_A(n)) - (c + \frac{t(s^*_A(n))}{n})\right)dx\right] \tag{3.10}$$

$$= m(v-c) - \frac{5mt(s^*_A(n))}{4n}, \tag{3.11}$$

and total surplus is:

$$
\begin{aligned}
T^A(s_A^*(n), n) &= n\Pi(s_A^*(n), \tau) + C(s_A^*(n), \tau) \\
&= m(v-c) - \frac{mt(s_A^*(n))}{4n} - nF(s_A^*(n), \tau).
\end{aligned} \tag{3.12}
$$

Proposition 2 examines the effects of changing the number of firms n on the equilibrium choices of scope, price, profits and surplus:

Proposition 2. *As the number of firms n increases*

(a) The equilibrium level of scope $s_A^*(n)$ decreases:

$$
\frac{ds_A^*(n)}{dn} = \frac{2mt_1(s_A^*(n))}{n[4n^2F_{11}(s_A^*(n), \tau) + mt_{11}(s_A^*(n))]} < 0, \tag{3.13}
$$

(b) Equilibrium prices $p_A^*(n)$ decrease:

$$
\frac{dp_A^*(n)}{dn} = \frac{t_1(s_A^*(n))}{n}\frac{ds_A^*(n)}{dn} - \frac{t(s_A^*(n)}{n^2} < 0 \tag{3.14}
$$

(c) Equilibrium net profits $\Pi^A(s_A^*(n), n)$ decrease:

$$
\frac{d\Pi^A(s_A^*(n), n)}{dn} = -\frac{2mt(s_A^*(n))}{n^3} + \left(\frac{mt_1(s_A^*(n))}{n^2} - F_1(s_A^*(n), \tau)\right)\frac{ds_A^*(n)}{dn} < 0, \tag{3.15}
$$

(d) Consumer surplus $C^A(s_A^*(n), n)$ increases:

$$
\frac{dC^A(s_A^*(n), n)}{dn} = \frac{5mt(s_A^*(n))}{4n^2} - \frac{5mt_1(s_A^*(n))}{4n}\frac{ds_A^*(n)}{dn} > 0 \tag{3.16}
$$

(e) Total surplus $T^A(s_A^*(n), n)$ increases if:

$$
\frac{mt(s_A^*(n))}{4n^2} > F(s_A^*(n), \tau) \tag{3.17}
$$

The increase in the number of firms n has a two-fold effect on equilibrium price. The second term on the RHS of equation (3.14) is the direct effect of increasing the number of firms on price, which is the only effect in the usual model with exogenous scope, and is always negative. On the other hand, the first term on the RHS of equation (3.14) highlights the value to firms of being able to strategically alter their product scope, which results in a positive price adjustment on account of the equilibrium reduction in scope. Proposition 2(b) tells us that under the model's convexity assumptions about $t(s)$, the negative effect always dominates the positive one, and therefore, having a higher number of firms does lead to lower prices.

Similar results are established for firm profitability and consumer surplus in parts (c) and (d) of the proposition. From (3.15) and (3.16), it is evident that the ability to strategically alter product scope mitigates both the decrease in firm profitability as well as the increase in consumer surplus that accompany an increase in the number of incumbent firms. In each of these expressions, the firms term provides the change in the corresponding outcome (profit and consumer surplus respectively) when scope is exogenous, while the second term, which is opposite in sign to the overall derivative, represents the moderating effect of product scope.

The next proposition characterizes how progress in technology or an expansion in market size affect equilibrium pricing and scope. Intuitively, since $F_2(s,\tau) < 0$ and $F_{12}(s,\tau) < 0$, the fixed cost as well as the 'marginal fixed cost' curve $F_1(s,\tau)$ are lower everywhere as τ increases and hence one would expect scope to increase.

Proposition 3. *(a) Technology progress (an increase in τ) results in an increase in the equilibrium level of product scope and a reduction in equilibrium price.*

(b) An increase in market size m results in a higher equilibrium level of product scope and a lower equilibrium price.

Notice from (3.8) that the symmetric equilibrium price is not directly dependent on either τ or m. The price reductions that accompany an improvement in the state of technology or increase in market size are entirely indirect, caused by the equilibrium increase in scope. An implication of Proposition 3 is that enabling incumbent firms to expand sales of their products to a new market (for instance, by including consumers in a different geographical location) results in an increase in product scope and a reduction in price for all consumers, including those in the *existing* market. It also simultaneously increases firm profits.

3.3. Entry-deterring oligopoly

This section characterizes equilibrium entry deterring behavior by n incumbent oligopolists. The sequence of events in this game is as follows: In stage 1, the incumbents choose symmetric levels of product scope. In stage 2, a different set of n firms from an adjacent oligopolistic industry evaluate potential entry into the first industry, and enter if it is profitable to do so at a level of scope identical to that of the incumbents. In stage 3 all firms choose prices with complete knowledge of the number of firms in the industry and product scope. Finally, based on the product offerings and prices, consumers make their purchase decisions, and the firms receive their payoffs.

We restrict the incumbent firms to choosing symmetric levels of scope; this is analogous to this choice of fixed cost of entry being an 'industry-wide' entry deterring decision (as in Spence, 1977). Further, we constrain entrants to choosing the same value of product scope as the incumbents, if they enter. If the n potential entrants do in fact enter, this results in a total of $2n$ symmetrically located firms. Under the assumption that entry is not blockaded (that is, that the symmetric n-firm equilibrium choices $p_A^*(n), s_A^*(n)$ do not naturally deter entry), the symmetric entry-deterring choices of product scope and price are specified in Proposition 4.

Proposition 4. *The symmetric n-firm entry-deterring choices* $s_D^*(n)$, $p_D^*(n)$ *of product scope and price are given by:*

$$F(s_D^*(n), \tau) = \frac{mt(s_D^*(n))}{4n^2}; \tag{3.18}$$

$$p_D^*(n) = c + \frac{t(s_D^*(n))}{n}. \tag{3.19}$$

Net profits for incumbents under entry deterrence are:

$$\Pi^D(s_D^*(n), n) = \frac{mt(s_D^*(n))}{n^2} - \frac{mt(s_D^*(n))}{4n^2} = \frac{3mt(s_D^*(n))}{4n^2}, \tag{3.20}$$

consumer surplus is:

$$C^D(s_D^*(n), n) = m(v - c) - \frac{5mt(s_D^*(n))}{4n}, \tag{3.21}$$

and total surplus is:

$$T^D(s_D^*(n), n) = m(v - c) - \frac{mt(s_D^*(n))}{2n}. \tag{3.22}$$

Comparative statics under entry deterrence are qualitatively similar to those established in Propositions 2. As the number of incumbent firms increases, there is a decrease in the level of product scope $s_D^*(n)$ that is required to deter entry. This makes intuitive sense, because a larger number of incumbents makes the industry more competitive, thereby reducing the potential gains from entry. Also, while the higher number of firms in the industry drives the equilibrium prices lower, this downward pressure on price is somewhat mitigated by the symmetric reduction in scope.

The sensitivity of entry deterring scope $s_D^*(n)$ and the accompanying equilibrium price to changes in technology τ, and to changes in market size m are also directionally similar to the results of Proposition 3. Thus as τ or m increases, the level of product scope required to deter entry increases, and the symmetric equilibrium price decreases. An intuitive explanation is that as technology progresses and τ increases, fixed costs fall for all levels of scope (because $F_2(s, \tau) < 0$). As a consequence, a higher level of scope is necessary in order to raise fixed costs to the level where entry is deterred. Similarly, as the market size m increases, so does the revenue opportunity of entering, warranting an increase in scope required to deter entry. The analytical details are available on request.

3.4. Comparison of outcomes

We now compare the outcomes of Propositions 1 and 4. If entry is not blockaded by a symmetric choice of $s_A^*(n)$, this implies that entrants will be able to make positive profits by offering this level of scope upon entry, which in turn implies that:

$$\frac{mt(s_A^*(n))}{4n^2} - F(s_A^*(n), \tau) > 0. \tag{3.23}$$

This implies that:

$$F(s_A^*(n), \tau) < \frac{mt(s_A^*(n))}{4n^2} \tag{3.24}$$

From (3.18), we know that at the entry deterring level of product scope $s_D^*(n)$, $F(s_D^*(n), \tau) = \frac{mt(s_D^*(n))}{4n^2}$. Therefore since $F_1(s, \tau) > 0$, we can conclude from (3.24) that

$$s_D^*(n) > s_A^*(n), \tag{3.25}$$

and additionally from (3.25), (3.8) and (3.19) that:

$$p_D^*(n) < p_A^*(n). \tag{3.26}$$

Intuitively, since entry is not blockaded at $s_A^*(n)$, a higher level of product scope is needed to deter entry. The higher level of product scope increases fixed costs and at the same time decreases the equilibrium prices, thus decreasing potential profits and making entry unattractive. The next proposition compares profits and surplus under the two scenarios and contributes towards understanding the welfare effects of bilateral oligopolistic convergence discussed in section 4.

Proposition 5. *In a market with n symmetrically located firms, if entry is not blockaded at the oligopolistic equilibrium level of product scope, then*

(a) Firm profits are higher under the oligopoly equilibrium as compared to those under entry deterrence :

$$\Pi^A(s_A^*(n), n) > \Pi^D(s_D^*(n), n). \tag{3.27}$$

(b) Consumer surplus is lower under the oligopoly equilibrium, as compared to that under entry deterrence:

$$C^A(s_A^*(n), n) < C^D(s_D^*(n), n). \tag{3.28}$$

(c) Total surplus is higher under the oligopoly equilibrium, as compared to that under entry deterrence:

$$T^A(s_A^*(n), n) > T^D(s_D^*(n), n). \tag{3.29}$$

While we have characterized and discussed the market outcomes in a scenario where entry is successfully deterred, we have not in fact established whether it is optimal for the incumbents to deter entry. It is possible that they may actually be better off just accommodating the entrants rather than keeping them out. This issue is discussed in the following section.

4. Bilateral entry in converging technology markets

This section uses the results of section 3 to analyze a model of converging technology markets. As discussed in Section 1, this model is motivated by the observation that in many information technology industries, the primary threat of entry is from existing firms in related industries (rather than new start-up firms), and is often triggered by strategic responses to technological progress that makes mobility across industry boundaries feasible. Besides, in technology markets, this movement is often bilateral: firms in a pair of adjacent industries threaten to enter each others' core markets.

4.1. Sequence and timing of events

There are two industries 1 and 2, each of which consists of n incumbent firms, and each of which has the demand and cost structure described in section 2. Consumers in each market are assumed to be distinct. Apart from the firms in these two industries, we assume that there are no other potential entrants. The sequence of events is as follows: In the first stage, firms in each industry choose the levels of scope for the products in their own markets. In the second stage, with complete information about the first-stage actions, firms in each industry decide whether or not to enter the other market. If entry occurs, then entrants are restricted to choosing the same product scope as the incumbents. In stage three, active firms in each market choose prices in a non-cooperative fashion. Finally, based on the prices and levels of scope in each market, consumers in each market make their purchase decisions, and firms receive their payoffs.

4.2. Firm decisions and payoffs

Firms make two sets of decisions before choosing prices. In the first stage, firms choose product scope in their own industry, towards either trying to deter entry (D) in the later stage, or towards trying to accommodate entry (A) in the later stage. In the second stage, contingent on the decisions made by the firms in the other market, and their own first stage decisions, each firm decides whether to stay out (S) of the other industry, or whether to enter (E) the other industry .

In each industry, we assume that the decisions made by each firm are symmetric. That is, each firm chooses the same level of product scope and price, and also chooses the same deter/accommodate and enter/stay out decision. However, the symmetric decision can be different

	Industry 1 actions	**Industry 2 actions**	**Industry 1 payoff**
$DSDS$	Deter, Stay out	Deter, Stay out	$\Pi^D(s_D^*(n), n)$
$DSDE$	Deter, Stay out	Deter, Enter	0
$DEDS$	Deter, Enter	Deter, Stay out	$\Pi^D(s_D^*(n), n)$
$DEDE$	Deter, Enter	Deter, Enter	0
$ASAS$	Accommodate, Stay out	Accommodate, Stay out	$\Pi^A(s_A^*(2n), n)$
$ASAE$	Accommodate, Stay out	Accommodate, Enter	$\Pi^A(s_A^*(2n), 2n)$
$AEAS$	Accommodate, Enter	Accommodate, Stay out	$\Pi^A(s_A^*(2n), n) + \Pi^A(s_A^*(2n), 2n)$
$AEAE$	Accommodate, Enter	Accommodate, Enter	$2\Pi^A(s_A^*(2n), 2n)$
$DSAS$	Deter, Stay out	Accommodate, Stay out	$\Pi^D(s_D^*(n), n)$
$DSAE$	Deter, Stay out	Accommodate, Enter	0
$DEAS$	Deter, Enter	Accommodate, Stay out	$\Pi^D(s_D^*(n), n) + \Pi^A(s_A^*(2n), 2n)$
$DEAE$	Deter, Enter	Accommodate, Enter	$\Pi^A(s_A^*(2n), 2n)$
$ASDS$	Accommodate, Stay out	Deter, Stay out	$\Pi^A(s_A^*(2n), n)$
$ASDE$	Accommodate, Stay out	Deter, Enter	$\Pi^A(s_A^*(2n), 2n)$
$AEDS$	Accommodate, Enter	Deter, Stay out	$\Pi^A(s_A^*(2n), n)$
$AEDE$	Accommodate, Enter	Deter, Enter	$\Pi^A(s_A^*(2n), 2n)$

Table 4.1: Summary of payoffs under each combination of actions

across the two industries. As in section 3, we focus on cases for which entry is not blockaded in either industry.

Under these assumptions, the choices of product scope and price are governed by the equilibria described in Propositions 1 and 4. Therefore, the payoffs to each sequence of decisions can be derived according to the equilibrium profit functions Π^A and Π^D, adjusting the total number of firms in each industry based on whether firms from the other industry have entered or not. This leads to the following components of the final payoffs:

(a) If firms in an industry choose product scope to deter entry (D), and entry is successfully deterred (S), each incumbent firm gets a payoff of $\Pi^D(s_D^*(n), n)$ from that industry. This is the payoff from the entry-deterring choice of product scope of the n-firm oligopoly (section 4). Note that D is the choice made by the firm's own industry, and S is a choice made by firms in the other industry.

(b) If firms in an industry choose product scope to deter entry (D), but entry occurs (E), then each incumbent and entrant firm gets a payoff of zero from that industry. This because the equilibrium choice of scope $s_D^*(n)$ that deters entry is chosen to induce a payoff of zero for a potential entrant who enters.

(c) If firms in an industry choose product scope to accommodate entry (A) and entry occurs

(E), then each incumbent and entrant firm gets a payoff of $\Pi^A(s_A^*(2n), 2n)$, which is the equilibrium payoff from the $2n$-firm oligopoly without an entry threat.

(d) If firms in an industry choose product scope to accommodate entry (A) but entry does not occur (S), then each incumbent firm gets a payoff of $\Pi^A(s_A^*(2n), n)$ from their industry. This is the equilibrium payoff from the n-firm oligopoly, when scope is $s_A^*(2n)$, or at the level chosen for equilibrium with $2n$ firms[3].

The payoffs from each combination of decisions, from the point of view of firms in one of the two symmetric industries, are summarized in Table 1. A comprehensive explanation of each of these payoffs is cumbersome, so we describe just a few illustrative cases. Under the first combination of actions $DSDS$, there are n firms in industry 1, each of whom has chosen to deter entry, and each of whom have chosen not to enter industry 2. As a consequence, each firm gets the n-firm entry-deterring payoff $\Pi^D(s_D^*(n), n)$ from industry 1, and gets no payoff from industry 2, for a total payoff of $\Pi^D(s_D^*(n), n)$. Under the second combination of actions $DSDE$, on the other hand, while firms in industry 1 have chosen to stay out of industry 2 and to deter entry in industry 1, the firms in industry 2 have chosen to enter industry 1. Consequently, the firms in industry 1 get no payoff from their own industry (since entry has occurred by n firms, all firms get zero payoff), and no payoff from industry 2 (since they have chosen to stay out).

Similarly, under the combination $DEAS$, firms in industry 1 successfully deter entry from their own industry (payoff of $\Pi^D(s_D^*(n), n)$), and enter industry 2, where they are accommodated (payoff of $\Pi^A(s_A^*(2n), 2n)$). Under the combination $ASDE$, the firms accommodate entry in their own industry (payoff of $\Pi^A(s_A^*(2n), 2n)$), and stay out of industry 2 (payoff of zero), for a total payoff of $\Pi^A(s_A^*(2n), 2n)$. Similar reasoning yields the payoffs for all the other combinations listed in Table 1.

[3] One could argue that under a set of actions in which firms choose to accomodate in an industry, but entry does not occur, the equilibrium payoff should be $\Pi^A(s_A^*(n), n)$ – simply the n-firm oligopoly payoff, rather than the higher value $\Pi^A(s_A^*(2n), n)$, since $\Pi^A(s_A^*(2n), n)$ is not a Nash equilibrium payoff. However, this would be inconsistent with the firm making their price and scope choices prior to knowing whether entry has occured. As it turns out, this does not affect the results – this outcome is never on the subgame perfect equilibrium path, and under either assumption (or any convex combination thereof), the actual equilibrium remains unchanged.

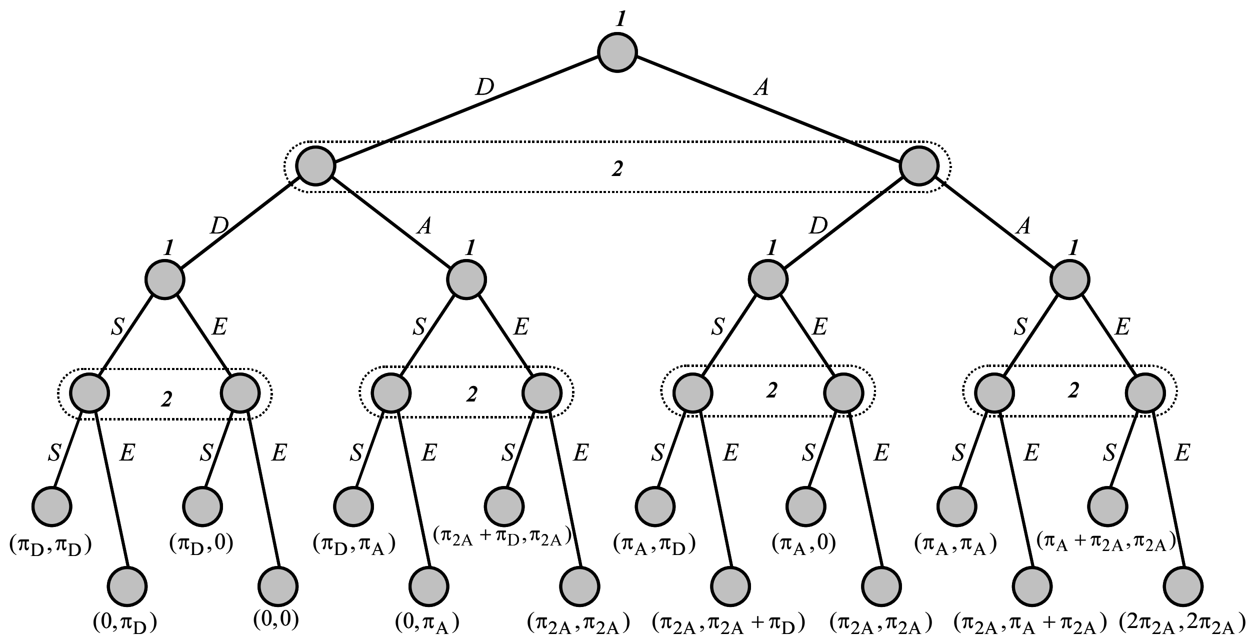

Figure 4.1: Game tree for the bilateral oligopolistic entry deterrence game, for one representative firm from each industry.

4.3. Equilibrium

Having specified the payoffs to the firms under each set of actions, we now specify the subgame perfect Nash equilibria of the bilateral entry game. Its extensive form and payoffs are shown in Figure 4.1, for a representative player from each industry.

When choosing equilibria in the subgames, we assume that if a player is indifferent between E and S (that is, the payoffs from entering and from staying out are equal), then the player chooses S (to stay out)[4].

Proposition 6. *The bilateral entry game has a unique subgame perfect Nash equilibrium.*

(a) If $\Pi^D(s_D^*(n), n) \geq \Pi^A(s_A^*(2n), 2n)$*, then the equilibrium strategies of all firms are DS (deter, stay out), the equilibrium choice of scope is* $s_D^*(n)$*, the equilibrium prices are* $p_D^*(n)$*, and the equilibrium payoffs to each firm are* $\Pi^D(s_D^*(n), n)$*.*

(b) If $\Pi^D(s_D^*(n), n) < \Pi^A(s_A^*(2n), 2n)$*, then the equilibrium strategies of all firms are AE*

[4] This simply defines equilibrium outcomes for knife's edge cases. One could interpret this as implicitly assuming a small cost of mobility across industries, which would imply that unless profits from entering are strictly higher, the player does not enter. We do not explicitly specify such a cost, however, since it would then affect the optimal choice of entry-deterring scope, thereby complicating the analysis substantially.

		Industry 2	
		Deter	*Accommodate*
Industry 1	*Deter*	$\Pi^D(s_D^*(n),n), \Pi^D(s_D^*(n),n)$	$\Pi^D(s_D^*(n),n)+\Pi^A(s_A^*(2n),2n)$, $\Pi^A(s_A^*(2n),2n)$
	Accommodate	$\Pi^A(s_A^*(2n),2n)$, $\Pi^D(s_D^*(n),n)+\Pi^A(s_A^*(2n),2n)$	$2\Pi^A(s_A^*(2n),2n)$, $2\Pi^A(s_A^*(2n),2n)$

Table 4.2: Nash payoffs from second-stage subgames, for each of the first stage actions

(accommodate, enter), the equilibrium choice of scope is $s_A^*(2n)$*, the equilibrium prices are* $p_A^*(2n)$*, and the equilibrium payoffs to each firm are* $2\Pi^A(s_A^*(2n),2n)$.

The derived payoff matrix for the first stage of the game after computing the Nash equilibrium outcomes of the second stage subgames is summarized in Table 4.2. When $\Pi^D(s_D^*(n),n) < \Pi^A(s_A^*(2n),2n)$, the equilibrium outcome of the game is Pareto-efficient, since the payoffs to the firms are higher at the equilibrium outcome than in any other feasible outcome. On the other hand, if $\Pi^A(s_A^*(2n),2n) \leq \Pi^D(s_D^*(n),n) < 2\Pi^A(s_A^*(2n),2n)$, the game becomes similar to a one-shot prisoners dilemma. Both firms would be better off under the Accommodate-Accommodate outcome, but since Deter is a dominant strategy for both players, they end up at the inefficient entry-deterring outcome.

4.4. Technological progress and equilibrium changes

Proposition 6 shows that the relative magnitudes of the n-firm entry-deterring equilibrium profits $\Pi^D(s_D^*(n),n)$ and the $2n$-firm standard oligopoly profits $\Pi^A(s_A^*(2n),2n)$ play the crucial role in determining the equilibrium outcome of the bilateral entry game. As τ increases, both these profit functions tend to decrease. If one decreases more rapidly than the other, this can cause a shift from one equilibrium outcome to another, resulting in a significant change in industry concentration and investment in product scope, and a redistribution of surplus across firms and consumers. In this section, we discuss two possible cases where this occurs.

The first case is illustrated in Figure 4.2(a), and represents a situation in which technological progress has a higher impact on the entry-deterring profits. Both $\Pi^D(s_D^*(n),n)$ and $\Pi^A(s_A^*(2n),2n)$ are decreasing as τ increases. While $\Pi^D(s_D^*(n),n)$ starts out higher (indicating the optimality of entry deterrence at lower levels of technology τ), it decreases more rapidly than $\Pi^A(s_A^*(2n),2n)$. At a critical point τ^*, the profit functions cross, after which accommodating entry is optimal, and

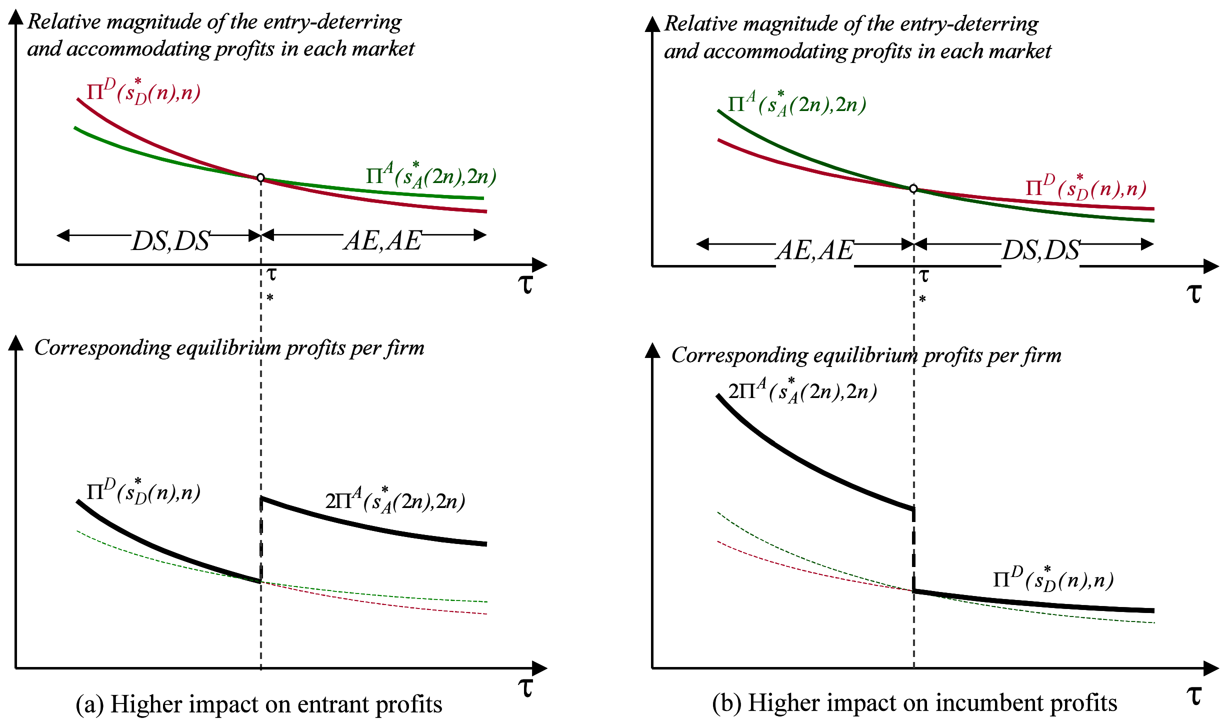

Figure 4.2: Illustrates possible discontinuous shifts in total per-firm profits as the relative magnitude of the entry-deterring and accomodating profits in *each* industry vary on account of technological progress.

$\Pi^A(s_A^*(2n),2n) > \Pi^D(s_D^*(n),n)$. While the progress in τ and the changes in these two functions are gradual, the changes in firm profits are substantial and discontinuous, since the equilibrium outcome now shifts to $AEAE$, resulting in a doubling of firm profits and and the number of firms in both industries. Moreover, there is an accompanying substantial drop in product scope, and a corresponding drop in fixed costs. In addition, since the equilibrium shifts from an entry-deterring one, to a standard $2n$ firm oligopoly, consumer surplus is likely to drop substantially.

The second case, illustrated in Figure 4.2(b), is where technological progress has a greater impact on the $2n$-firm oligopoly profits. Again, both $\Pi^D(s_D^*(n),n)$ and $\Pi^A(s_A^*(2n),2n)$ are decreasing as τ increases, but in this case, $\Pi^A(s_A^*(2n),2n)$ starts out higher, and decreases more rapidly, until the curves cross at some point. Consequently, while entry would have been accommodated initially, entry deterrence now becomes the equilibrium strategy for incumbents. Profits fall substantially, as firms recede into their core industries, and raise their investments in product scope, so as to deter entry. However, consumer surplus rises sharply, as the value of individual products increases, and prices drop. While it is technologically feasible (and bilaterally profit improving) to accommodate entry in both markets, it is no longer strategically possible to do so.

The interesting aspect of both these cases is that while technological progress leads to 'innovation' of sorts in both scenarios, the outcomes for firms and consumers are starkly different. In the first case, when it is accompanied by an expansion by firms into new markets, and increase in the number of firms in both industries, consumers paradoxically suffer on account of technological progress. In the latter case, where there is focused and high individual investment by each firm in their core markets, albeit at a level that is socially less efficient, consumers nevertheless benefit substantially.

5. Discussion and conclusions

Technological progress often leads firms to compete in each others' industries. Though this has been highlighted recently by digital convergence and the sudden increase in products and services that span traditionally distinct industry boundaries, it is not a new occurrence in technology markets. For instance, Breshanan and Greenstein (1999) talk about the 'competitive crash' in the computer industry in the early 1990's, when, as described in their paper, seller rents were dramatically reallocated across market segments, and firms that had previously supplied different segments now competed for the same consumers.

This 'competitive crash' was not preceded by a technological shock, and for the most part, neither has the current trend towards product convergence. Our model support these observations, providing an explanation of how gradual progress in technology may lead to cycles in which there are periods of gradual price and profit declines, followed by sudden changes as firms cross industry boundaries. The sudden changes occur when the equilibrium shifts to one of entry accommodation. Immediately following this shift, if entry is blockaded, there is a period of relative 'calm', after which technology progresses to the point where it becomes necessary to deter entry in one's markets again. The change at this point, and following it, are still gradual, until technology progresses to the point where accommodation becomes optimal again, thereby causing another substantial industry realignment.

Independently, our analysis of oligopoly with endogenous scope and the threat of entry has yielded a number of interesting results. We have shown that when firms in technology markets are able to respond to changes in industry concentration by adjusting both prices and product scope, equilibrium reductions in scope mitigate the price and profit reductions that would accompany an increase in the number of incumbent firms. This adjustment leads to a lower increase in consumer surplus, than would have happened in an industry where product scope was exogenously specified. Moreover, when firms use product scope to deter entry, their choices of scope are socially less efficient (they are excessive) than their corresponding choices when they accommodate entry, although in the former case, the consumers are better off. As technology progresses, the response by entry deterring firms is to further increase product scope, thereby often reducing their own profits, and continuing an inefficient transfer of surplus to consumers. This might form the basis for one explanation of the observed long-term trends of hedonic price reductions in technology markets. In this context, encouraging entry-deterring behavior under the argument that it benefits consumers is unlikely to be good long-term policy. However, if a policy maker were to attempt to rectify this inefficiency (by mandating a level of scope through imposition of *dejure* industry standards, for instance, while still letting firms to compete on price), this is bound to reduce consumer surplus. As a consequence, regulatory action that is socially optimal is unlikely to be politically viable, and vice versa.

An increase in the total market size for the product results in an increase in product scope and a reduction in prices for all consumers, including those in the existing market. This is consistent with firms being able to spread their fixed costs of scope over a higher number of consumers – consequently, they increase scope, and reduce prices in response. For instance, if wireless technology developed for a national market were compatible with the standards in other national markets, this would translate into gains not just for the manufacturers of wireless handsets and communications equipment, but also for consumers in this market, since they would benefit from significantly better products in their own market, at a lower price. Consequently, government regulatory policy that encourages (or mandates) shared standards, even at the cost of mandating that firms invest more in product design and software so as to cater to a multinational audience, will lead to substantial consumer benefits, and will do so in a manner that improves firm profits. This may be instructive for markets like the United States, which have chosen a purely industry-driven approach to standards setting for cellular telephony.

A limitation of the model is its interpretation of a dynamic phenomenon based on comparative statics, which precludes dynamic strategic choices by firms which anticipate technological progress, inherently assuming that they make myopic choices. Formalizing our technology cycles in a dynamic model is a promising direction for future research. Other natural extensions include allowing asymmetric industry concentration and market size in the two converging industries, so as to enable the analysis of mobility decisions when one firm has more to gain from entry, and studying the effects of heterogeneity in responses to technological progress, with possible information asymmetry or technology spillovers.

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A. Appendix: Proofs

Proof of Proposition 1

Suppose each firm chooses product scope choice s in the first stage. Also assume that all firms except firm i choose price p in the second stage. If firm i chooses price p_i, then based on equation

(3.6), its payoff will be:

$$\Pi(p_i, s|p, s, p, s) = 2m(p_i - c)\left(\frac{p - p_i + \frac{1}{n}t(s)}{2t(s)}\right) - F(s, \tau). \tag{A.1}$$

The payoff (A.1) is strictly concave in price p_i, and hence the first order conditions are both necessary and sufficient to yield a unique best response p_i^* for firm i, which is given by:

$$\Pi_1(p_i^*, s|p, s, p, s) = \frac{m}{t}\left(p - 2p_i^* + \frac{1}{n}t(s) + c\right) = 0. \tag{A.2}$$

We can now set $p_i^* = p$ in equation (A.2) to obtain the following unique symmetric price equilibrium following a symmetric choice of scope by all firms, which is given by

$$p^* = c + \frac{t(s)}{n}. \tag{A.3}$$

Characterizing equilibrium symmetric product scope is less straightforward. We proceed as follows: first, we find the marginal change in profit that a firm gets from a small unilateral deviation in product scope from the symmetric level and set this profit to zero. This is done under the restriction that firms' choice of prices in the second stage will be adjusted appropriately to constitute a Nash equilibrium under the new set of product scopes.

Assume that all firms choose a symmetric level of product scope s in the first stage, followed by the symmetric Nash equilibrium price p (given by equation (A.3)) in the second. Now assume that firm i deviates from this symmetric level of first stage scope by a small amount. By choosing a different level of product scope, firm i alters the second stage price equilibrium. Therefore, firm i's marginal change in profits from a small variation in s_i is:

$$\frac{d\Pi_i}{ds_i} = \frac{d\pi_i}{ds_i} - F_1(s_i, \tau), \tag{A.4}$$

with $\frac{d\pi_i}{ds_i}$ given by:

$$\frac{d\pi_i}{ds_i} = \frac{\partial\pi_i}{\partial s_i} + \frac{\partial\pi_i}{\partial p_i}\frac{dp_i^*}{ds_i} + \frac{\partial\pi_i}{\partial p_{i+1}}\frac{dp_{i+1}^*}{ds_i} + \frac{\partial\pi_i}{\partial p_{i-1}}\frac{dp_{i-1}^*}{ds_i}, \tag{A.5}$$

where the subscripts i, $i-1$ and $i+1$ represent the firm i and its neighbors. The first term in equation (A.5) is the direct effect of increasing scope on gross profit, without taking the second stage adjustments in price into consideration. The p^* terms represent the firms' best responses in the second stage of the game, given the choices of product scope in stage 1. Since firms' payoff functions are strictly concave in their own prices in the second stage, the p^* terms should satisfy the n first order conditions for the second stage and ensure that equation (A.4) takes the second stage equilibrium adjustments in price into consideration.

The second term in equation (A.5) is zero, since $\frac{\partial \pi_i}{\partial p_i} = 0$ under subgame perfection. Further, the third and fourth terms are identical, owing to the symmetric choices by all firms except firm i. We will focus only on the $i+1$ terms.

Partially differentiating equation (3.5) with respect to s_i after substituting q_i from equation (3.4), we have:

$$\frac{\partial \pi_i}{\partial s_i} = -m(p_i - c)\left(\frac{\left(p_{i-1} - p_i + \frac{1}{n}t(s_{i-1})\right)t_1(s_i)}{\left(t(s_i) + t(s_{i-1})\right)^2} + \frac{\left(p_{i+1} - p_i + \frac{1}{n}t(s_{i+1})\right)t_1(s_i)}{\left(t(s_i) + t(s_{i+1})\right)^2}\right). \tag{A.6}$$

Evaluating (A.6) at the symmetric levels of product scope s and price p according to (A.3) yields:

$$\frac{\partial \pi_i}{\partial s_i} = -\frac{m t_1(s)}{2n^2}. \tag{A.7}$$

Partially differentiating both sides of equation (3.5) with respect to p_{i+1} after substituting the value of q_i from equation (3.4), we have:

$$\frac{\partial \pi_i}{\partial p_{i+1}} = m(p_i - c)\left(\frac{1}{t(s_i) + t(s_{i+1})}\right). \tag{A.8}$$

Evaluating (A.8) at the symmetric levels of product scope s and price p from (A.3) yields:

$$\frac{\partial \pi_i}{\partial p_{i+1}} = \frac{m}{2n}. \tag{A.9}$$

To evaluate $\frac{dp^*_{i+1}}{ds_i}$ in equation (A.5), we need to examine how the solution to the system of equations represented by the n first order conditions in the second stage responds to a change in s_i. This is hard to do for a generic n. We therefore follow Hendel and Figueiredo (1997) and use a first order approximation given by:

$$\frac{dp^*_{i+1}}{ds_i} = \frac{\partial p^*_{i+1}}{\partial s_i} + \frac{\partial p^*_{i+1}}{\partial p_i}\frac{dp^*_i}{ds_i}. \tag{A.10}$$

Under this approximation, the second stage first order condition for firm i, based on equation (3.5) is:.

$$\begin{aligned} \frac{d\pi_i}{dp_i} \quad | \quad p^*_i = m & \left(\frac{p_{i-1} - p^*_i + \frac{1}{n}t(s_{i-1})}{t(s_i) + t(s_{i-1})} + \frac{p_{i+1} - p^*_i + \frac{1}{n}t(s_{i+1})}{t(s_i) + t(s_{i+1})} \right) \\ & -m(p^*_i - c)\left(\frac{1}{t(s_i) + t(s_{i-1})} + \frac{1}{t(s_i) + t(s_{i+1})} \right), \end{aligned} \tag{A.11}$$

which should be equal to zero. Differentiating (A.11) with respect to s_i, rearranging and substituting the symmetric levels of product scope s and price p yields:

$$\frac{t_1(s)}{2nt(s)} - \frac{1}{t(s)}\frac{dp^*_i}{ds_i} - \frac{t_1(s)}{2nt(s)} = 0, \tag{A.12}$$

which implies that

$$\frac{dp^*_i}{ds_i} = 0. \tag{A.13}$$

Now, using $i+1$ in place of i in equations (3.5) and (3.4), and writing down the first order condition for the second stage of firm $i+1$, we have:

$$\begin{aligned} \frac{d\pi_{i+1}}{dp_{i+1}} \quad | \quad p^*_{i+1} = m & \left(\frac{p_i - p^*_{i+1} + \frac{1}{n}t(s_i)}{t(s_{i+1}) + t(s_i)} + \frac{p_{i+2} - p^*_{i+1} + \frac{1}{n}t(s_{i+2})}{t(s_{i+1}) + t(s_{i+2})} \right) \\ & -m(p^*_{i+1} - c)\left(\frac{1}{t(s_{i+1}) + t(s_i)} + \frac{1}{t(s_{i+1}) + t(s_{i+2})} \right), \end{aligned}$$

which should also be equal to zero. Differentiating with respect to s_i, rearranging and substituting the symmetric levels of product scope s and price p yields:

$$\frac{t_1(s)}{4nt(s)} - \frac{1}{t(s)}\frac{dp^*_{i+1}}{ds_i} = 0, \tag{A.14}$$

which implies that:

$$\frac{dp^*_{i+1}}{ds_i} = \frac{t_1(s)}{4n}. \tag{A.15}$$

Substituting from (A.7),(A.9), (A.10), (A.13) and (A.15) into equation (A.5) yields:

$$\frac{d\pi_i}{ds_i} = -\frac{mt_1(s)}{2n^2} + 2\left(\frac{m}{2n}\right)\left(\frac{t_1(s)}{4n}\right) = -\frac{mt_1(s)}{4n^2}. \tag{A.16}$$

Applying the Nash condition by substituting (A.16) into (A.4) and equating it to zero, we obtain the symmetric level of equilibrium product scope for the first stage of the game, which is given by the following condition.

$$F_1(s_A^*(n), \tau) = -\frac{mt_1(s_A^*(n))}{4n^2}, \tag{A.17}$$

which completes the proof.

Proof of Proposition 2

(a) Totally differentiating both sides of equation (A.17) with respect to n yields

$$F_{11}(s_A^*(n), \tau)\left(\frac{ds_A^*}{dn}\right) = -\frac{mt_{11}(s_A^*(n))}{4n^2}\left(\frac{ds_A^*}{dn}\right) + \frac{mt_1(s_A^*(n))}{2n^3}, \tag{A.18}$$

which rearranges to

$$\frac{ds_A^*(n)}{dn} = \frac{2mt_1(s_A^*(n))}{4n^3 F_{11}(s_A^*(n), \tau) + mnt_{11}(s_A^*(n))}. \tag{A.19}$$

Since $t_1(s) < 0$, $F_{11}(s, \tau) > 0$, and $t_{11}(s) > 0$, it follows that $\frac{ds_A^*(n)}{dn} < 0$.

(b) Totally differentiating both sides of equation (**??**) with respect to n yields

$$\frac{dp_A^*(n)}{dn} = \frac{t_1(s_A^*(n))}{n}\frac{ds_A^*(n)}{dn} - \frac{t(s_A^*(n))}{n^2}. \tag{A.20}$$

Substituting equation (A.19) into (A.20) yields:

$$\frac{dp_A^*(n)}{dn} = \frac{2m(t_1(s_A^*(n)))^2}{4n^4 F_{11}(s_A^*(n), \tau) + mn^2 t_{11}(s_A^*(n))} - \frac{t(s_A^*(n))}{n^2}, \tag{A.21}$$

which rearranges to:

$$\frac{dp_A^*(n)}{dn} = \frac{1}{n^2}\frac{m[2(t_1(s_A^*(n)))^2 - t_{11}(s_A^*(n))t(s_A^*(n))] - 4n^2 F_{11}(s_A^*(n), \tau)t(s_A^*(n))}{4n^2 F_{11}(s_A^*(n), \tau) + mt_{11}(s_A^*(n))}. \tag{A.22}$$

Under the convexity assumptions imposed on $t(s)$, we know that $2(t_1(s))^2 - t_{11}(s)t(s) \leq 0$. Since

$F_{11}(s,\tau) > 0$, this implies that the numerator of the RHS of equation (A.22) is strictly negative. The result follows.

(c) Totally differentiating both sides of equation (3.9) with respect to n yields

$$\frac{d\Pi^A(s_A^*(n),n)}{dn} = -\frac{2mt(s_A^*(n))}{n^3} + \left(\frac{mt_1(s_A^*(n))}{n^2} - F_1(s_A^*(n),\tau)\right)\frac{ds_A^*(n)}{dn}. \tag{A.23}$$

Substituting for $F_1(s_A^*(n),\tau)$ from (A.17) and $\frac{ds_A^*(n)}{dn}$ from (A.19) yields:

$$\frac{d\Pi^A(s_A^*(n),n)}{dn} = -\frac{2mt(s_A^*(n))}{n^3} + \left(\frac{mt_1(s_A^*(n))}{n^2} + \frac{mt_1(s_A^*(n))}{4n^2}\right)\left(\frac{2mt_1(s_A^*(n))}{4n^3F_{11}(s_A^*(n),\tau) + mnt_{11}(s_A^*(n))}\right). \tag{A.24}$$

Rearranging terms and simplifying, we obtain:

$$\frac{d\Pi^A(s_A^*(n),n)}{dn} = \frac{2m}{n^3}\left(\frac{4m\left[2(t_1(s_A^*(n)))^2 - t(s_A^*(n))t_{11}(s_A^*(n))\right] - 3m(t_1(s_A^*(n)))^2 - 16n^2F_{11}(s_A^*(n),\tau)t(s_A^*(n))}{4(4n^2F_{11}(s_A^*(n),\tau) + mt_{11}(s_A^*(n))}\right) \tag{A.25}$$

In (A.25) since $F_{11} > 0$ and $t_{11} > 0$, the denominator is positive, and since $2(t_1(s_A^*(n)))^2 - t(s_A^*(n))t_{11}(s_A^*(n)) \leq 0$, based on our convexity conditions for $t(s)$, the numerator is negative. The result follows.

(d) Totally differentiating both sides of equation (3.11) with respect to n yields:

$$\frac{dC^A(s_A^*(n),n)}{dn} = \frac{5mt(s_A^*(n))}{4n^2} - \frac{5mt_1(s_A^*(n))}{4n}\frac{ds_A^*(n)}{dn}. \tag{A.26}$$

Substituting from (A.19) for $\frac{ds_A^*(n)}{dn}$ and simplifying yields:

$$\frac{dC^A(s_A^*(n),n)}{dn} = \frac{5m}{4n^2}\left[\frac{4n^2F_{11}(s_A^*(n),\tau)t(s_A^*(n)) + m\left(t(s_A^*(n))t_{11}(s_A^*(n)) - 2(t_1(s_A^*(n)))^2\right)}{4n^2F_{11}(s_A^*(n),\tau) + mt_{11}(s_A^*(n)}\right]. \tag{A.27}$$

In (A.27) the denominator is positive since $F_{11} > 0$ and $t_{11} > 0$, and the numerator is also positive since $t(s_A^*(n))t_{11}(s_A^*(n)) - 2(t_1(s_A^*(n)))^2 \geq 0$. The result follows.

(e) Totally differentiating both sides of equation (3.12) with respect to n yields:

$$\frac{dT^A(s_A^*(n),n)}{dn} = \left(\frac{mt(s_A^*(n))}{4n^2} - F(s_A^*(n),\tau)\right) - \left(\frac{mt_1(s_A^*(n))}{4n} + nF_1(s_A^*(n),\tau)\right)\frac{ds_A^*(n)}{dn}. \quad \text{(A.28)}$$

Substituting for $F_1(s_A^*(n),\tau)$ from (A.17), we have:

$$\frac{dT^A(s_A^*(n),n)}{dn} = \left(\frac{mt(s_A^*(n))}{4n^2} - F(s_A^*(n),\tau)\right) - \left(\frac{mt_1(s_A^*(n))}{4n} - n\frac{mt_1(s_A^*(n))}{4n^2}\right)\frac{ds_A^*(n)}{dn}. \quad \text{(A.29)}$$

The second term equals zero and therefore (A.29) simplifies to

$$\frac{dT^A(s_A^*(n),n)}{dn} = \left(\frac{mt(s_A^*(n))}{4n^2} - F(s_A^*(n),\tau)\right). \quad \text{(A.30)}$$

The result follows.

Proof of Proposition 3

(a) Totally differentiating both sides of equation (A.17) with respect to τ yields:

$$F_{11}(s_A^*(n),\tau)\left(\frac{ds_A^*(n)}{d\tau}\right) + F_{12}(s_A^*(n),\tau) = -\frac{mt_{11}(s_A^*(n))}{4n^2}\left(\frac{ds_A^*(n)}{d\tau}\right), \quad \text{(A.31)}$$

which rearranges to:

$$\frac{ds_A^*(n)}{d\tau} = \frac{-F_{12}(s_A^*(n),\tau)}{F_{11}(s_A^*(n),\tau) + \frac{mt_{11}(s_A^*(n))}{4n^2}}. \quad \text{(A.32)}$$

Since $F_{12}(s,\tau) < 0$, $F_{11}(s,\tau) > 0$, and $t_{11}(s) > 0$, it follows that $\frac{ds_A^*(n)}{d\tau} > 0$. Next, totally differentiating both sides of equation (**??**) with respect to τ yields:

$$\frac{dp_A^*(n)}{d\tau} = \frac{t_1(s_A^*(n))}{n}\frac{ds_A^*(n)}{d\tau}, \quad \text{(A.33)}$$

and since $t_1(s) < 0$ and $\frac{ds_A^*(n)}{d\tau} > 0$, it follows that $\frac{dp_A^*(n)}{d\tau} < 0$.

(b) Totally differentiating both sides of equation (A.17) with respect to m yields:

$$F_{11}(s_A^*(n),\tau)\left(\frac{ds_A^*(n)}{dm}\right) = -\frac{t_1(s_A^*(n))}{4n^2} - \frac{mt_{11}(s_A^*(n))}{4n^2}\left(\frac{ds_A^*(n)}{dm}\right), \quad \text{(A.34)}$$

which yields:

$$\frac{ds_A^*(n)}{dm} = \frac{-t_1(s_A^*(n))}{4n^2 F_{11}(s_A^*(n), \tau) + m t_{11}(s_A^*(n))}. \tag{A.35}$$

Since $t_1(s) < 0$, $F_{11}(s, \tau) > 0$, and $t_{11}(s) > 0$, it follows that $\frac{ds_A^*(n)}{dm} > 0$. Totally differentiating both sides of equation (**??**) with respect to m yields:

$$\frac{dp_A^*(n)}{dm} = \frac{t_1(s_A^*(n))}{n} \frac{ds_A^*(n)}{dm} \tag{A.36}$$

Since $t_1(s) < 0$, we have $\frac{dp_A^*(n)}{dm} < 0$.

Proof of Proposition 4

Assume that each of the incumbents chooses a level of product scope s_D in stage 1 of the game. Given the restriction on the entrants' scope, they will also have to choose a level of product scope equal to s_D, should they decide to enter the market in stage 2. With symmetric levels of product scope and $2n$ firms in the market, we can use (A.3) to calculate the equilibrium prices in stage 3 as:

$$p_D = c + \frac{t(s_D)}{2n}. \tag{A.37}$$

Given symmetric choices of product scope and price (s_D, p_D), each firm receives a demand of $m/2n$. Thus the net profit for each entrant[5] is:

$$\Pi_E(p_D, s_D) = \frac{m t(s_D)}{4n^2} - F(s_D, \tau). \tag{A.38}$$

The entry deterring level of product scope follows from equating the entrants profits to zero in equation (A.38). This yields:

$$F(s_D^*(n), \tau) = \frac{m t(s_D^*(n))}{4n^2}. \tag{A.39}$$

Substituting the entry deterring level of scope in (A.3) yields the following equilibrium price:

$$p_D^*(n) = c + \frac{t(s_D^*)}{n}. \tag{A.40}$$

[5] If entry occurs, this will be the incumbents' profit as well.

Proof of Proposition 5

(a) In each of the two cases, given the appropriate level of scope s, the expression for net profits is

$$\Pi(s,n) = \frac{mt(s)}{n^2} - F(s,\tau). \tag{A.41}$$

Since $t_1(s) < 0$ and $F_1(s,\tau) > 0$, the expression above is strictly decreasing in s. Consequently, using $s_A^*(n) < s_D^*(n)$, the result follows.

(b) In each case, given the appropriate level of scope s, the expression for consumer surplus is

$$C(s,n) = m(v-c) - \frac{5mt(s)}{4n}. \tag{A.42}$$

Since $t_1(s) < 0$, the expression above is strictly increasing in s. Since $s_D^*(n) > s_A^*(n)$, the result follows.

(c) At a given level of scope s, the expression for total surplus is $C(s,n) + n\Pi(s,n)$. Thus,

$$T(s,n) = m(v-c) - \frac{mt(s)}{4n} - nF(s,\tau). \tag{A.43}$$

Differentiating both sides of (A.43) with respect to s yields:

$$T_1(s,n) = -\frac{mt_1(s)}{4n} - nF_1(s,\tau) \tag{A.44}$$

$$T_{11}(s,n) = \frac{-mt_{11}(s)}{4n} - nF_{11}(s,\tau). \tag{A.45}$$

Since $t_{11}(s) > 0$ and $F_{11}(s,\tau) > 0$ for all s, this establishes that $T(s,n)$ is strictly concave in s. Further, from (A.17) we know that $F_1(s_A^*(n),\tau) = -\frac{mt_1(s_A^*(n))}{4n^2}$, and therefore

$$T_1(s_A^*(n),n) = 0. \tag{A.46}$$

Since $s_A^*(n) < s_D^*(n)$, strict concavity of $T(s,n)$ implies that $T(s_A^*(n),n) > T(s_D^*(n),n)$.

Proof of Proposition 6

First, consider the payoff matrix for the second-stage subgame that follows a choice of Deter by firms in both industries (the DD subgame):

		Industry 2	
		Stay out (S)	*Enter* (E)
Industry 1	*Stay out* (S)	$\mathbf{\Pi}^D(\mathbf{s}_D^*(\mathbf{n}), \mathbf{n}),\ \mathbf{\Pi}^D(\mathbf{s}_D^*(\mathbf{n}), \mathbf{n})$	$0,\ \Pi^D(s_D^*(n), n)$
	Enter (E)	$\Pi^D(s_D^*(n), n),\ 0$	$0,\ 0$

Subgame DD

Clearly, S is a weakly dominant strategy for both players. Since we choose S over E when they yield the same payoffs, the Nash equilibrium of this subgame is SS.

Next, consider the payoff matrix for the DA subgame:

		Industry 2	
		Stay out (S)	*Enter* (E)
Industry 1	*Stay out* (S)	$\Pi^D(s_D^*(n), n),\ \Pi^A(s_A^*(2n), n)$	$0,\ \Pi^A(s_A^*(2n), n)$
	Enter (E)	$\mathbf{\Pi}^D(\mathbf{s}_D^*(\mathbf{n}), \mathbf{n}) + \mathbf{\Pi}^A(\mathbf{s}_A^*(\mathbf{2n}), \mathbf{2n}),$ $\mathbf{\Pi}^A(\mathbf{s}_A^*(\mathbf{2n}), \mathbf{2n})$	$\Pi^A(s_A^*(2n), 2n),\ \Pi^A(s_A^*(2n), 2n)$

Subgame DA

S is a weakly dominant strategy for firms in industry 2, and E is a dominant strategy for firms in industry 1. Consequently, the Nash equilibrium is ES.

Next, consider the payoff matrix for the AD subgame:

		Industry 2	
		Stay out (S)	*Enter* (E)
Industry 1	*Stay out* (S)	$\Pi^A(s_A^*(2n), n),\ \Pi^D(s_D^*(n), n)$	$\mathbf{\Pi}^A(\mathbf{s}_A^*(\mathbf{2n}), \mathbf{2n}),$ $\mathbf{\Pi}^D(\mathbf{s}_D^*(\mathbf{n}), \mathbf{n}) + \mathbf{\Pi}^A(\mathbf{s}_A^*(\mathbf{2n}), \mathbf{2n})$
	Enter (E)	$\Pi^A(s_A^*(2n), n),\ 0$	$\Pi^A(s_A^*(2n), 2n),\ \Pi^A(s_A^*(2n), 2n)$

Subgame AD

This is simply the DA payoff matrix transposed, with payoffs exchanged. In this case, S is a weakly dominant strategy for firms in industry 1, and E is a dominant strategy for firms in industry 2, which leads to the Nash equilibrium SE.

Finally, the payoff matrix for the AA subgame is:

		Industry 2	
		Stay out (S)	*Enter* (E)
Industry 1	*Stay out* (S)	$\Pi^A(s_A^*(2n), n),\ \Pi^A(s_A^*(2n), n)$	$\Pi^A(s_A^*(2n), 2n)$, $\Pi^A(s_A^*(2n), n) + \Pi^A(s_A^*(2n), 2n)$
	Enter (E)	$\Pi^A(s_A^*(2n), n) + \Pi^A(s_A^*(2n), 2n)$, $\Pi^A(s_A^*(2n), 2n)$	$\mathbf{2\Pi^A(s_A^*(2n), 2n),\ 2\Pi^A(s_A^*(2n), 2n)}$

Subgame AA

Clearly, E is a dominant strategy for both players, leading to the Nash equilibrium EE.

Therefore, under subgame perfection, the payoffs as seen by the players when making their stage 1 decisions are as follows:

		Industry 2	
		Deter	*Accommodate*
Industry 1	*Deter*	$\Pi^D(s_D^*(n), n), \Pi^D(s_D^*(n), n)$	$\Pi^D(s_D^*(n), n) + \Pi^A(s_A^*(2n), 2n)$, $\Pi^A(s_A^*(2n), 2n)$
	Accommodate	$\Pi^A(s_A^*(2n), 2n)$, $\Pi^D(s_D^*(n), n) + \Pi^A(s_A^*(2n), 2n)$	$2\Pi^A(s_A^*(2n), 2n), 2\Pi^A(s_A^*(2n), 2n)$

First stage payoffs, given Nash outcomes in the second stage

If $\Pi^D(s_D^*(n), n) > \Pi^A(s_A^*(2n), 2n)$, then Deter (D) is a dominant strategy for both players. On the other hand, if $\Pi^D(s_D^*(n), n) < \Pi^A(s_A^*(2n), 2n)$, then Accommodate (A) is a dominant strategy for both players. If $\Pi^D(s_D^*(n), n) = \Pi^A(s_A^*(2n), 2n)$, then any combination of actions is a Nash equilibrium. Consistent with our earlier assumption of firms choosing to stay out rather than enter, we choose DD as the outcome in this case (it is a knife's edge case and has no bearing on the subsequent discussion). The result follows.